A Method for Determining the Generators' Share in a Consumer Load

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***Abstract*—In the paper, a new method for determining the generators' contribution to a particular load is presented. The method uses the nodal generation distribution factors (NGDF-s). It features a search algorithm, capable of handling the active and reactive powers. The method has been tested on the IEEE 118 and on the Slovenian 275 bus systems. The results demonstrate that electrical energy flows from a producer to a customer based on physical rather than on economical rules. By means of the new method, it can be shown that in general, the customer does not obtain the full power from the selected power plant according to the contract but also from other power plants. The method of the NGDF-s can be used for transmission service pricing, for congestion management and for reactive power management.**

***Index Terms*—Load supplying, nodal generation distribution factor, power flow, power system.**

I. Introduction

In DEREGULATED power systems, new tools for control and operation are required. Among others, it is interesting to know, which power plants supply a particular customer and to what degree. In the paper, it is shown how this can be achieved by using the generation distribution factors.

The generation distribution factors have been developed for security and contingency analyzes. One of the most often used approaches to calculate them is the method of the generalized generation distribution factors (GGDF-s) [1]. They are based on a linearized DC model of the power system and describe the impact of every generator on the active power flow on a line.

In certain deregulated power systems, the GGDF-s are also used as a part of a tool for transmission service pricing based on the MW-MILE methodology. However, there is a question whether it is more correct to take into account the impact of a particular generator or its actual share in the line power flow [2].

As a result, some new approaches to generation distribution factors have been proposed, which determine the share of a particular generator in the line flow. The two of the most popular types among them are topological generation distribution factors (TGDF-s) [2]–[4] and factors based on the generator domains (DGDF-s) [5], [6]. In general, these two methods are more appropriate for the active than for the reactive power. The authors recently suggested some improvements of their methods to overcome this problem. Moreover, the DGDF method obtains the factors for the entire "common," i.e., the set of buses supplied from the same set of generators. These disadvantages have been the reason for developing a new method of generation distribution factors named the nodal generation distribution factors (NGDF-s). NGDF-s determine the share of a particular generator in every line flow. They can be used for both, the active and reactive power flows since the transmission losses are taken into account as well.

As mentioned, the information about the generator's contributions to a line flow can be used as a base for calculating the generator's contributions to a particular load. The new method based on the NGDF-s has been developed and tested on different power systems and for various operating states.

II. Generators' Share in a Bus Load

The method for determining contributions of generators to a particular bus load could be based on appropriate generation distribution factors. Although various methods for determining the generation distribution factors can be used, the new method of the nodal generation distribution factors (NGDF-s) has been developed for that purpose. Due to its features, it has been selected as the most appropriate. It determines the generator's contributions to the active and reactive power flows on a line accurately and in a simple way. The method of the NGDF-s is derived in the Appendix

In the procedure, the active or reactive powers are treated independently and calculation of their factors is made separately. In the subsequent calculation, the symbol M represents either the active (P) or the reactive (Q) power:

$$M \in \{P, Q\}. \tag{1}$$

Let the symbol $L_{ip,\,k}$ be the nodal generation distribution factor. Multiplied with the line flow (M_{ip}), it represents the share $(M_{ip,\,k})$ of the generator k to the power flow on the line ip as follows:

$$M_{ip,\,k} = L_{ip,\,k} M_{ip}. \tag{2}$$

The generator's share in supplying the load on the bus i could be obtained using the method described below.

The share $(M_{i,\,k})$ of the generator k supplying the bus i should take into account the line inflows and generated power at the bus i. For the reactive power, generators, sources of

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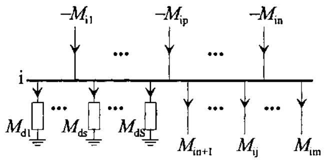

Fig. 1. Mixing of the electrical energy at the power system bus.

reactive power (capacitor banks) and transmission line charging are taken into account.

$$M_{i,k} = \begin{cases} -\sum\limits_{ip\in\Psi_i} L_{ip,k}M_{ip} + M_i; & k = i \\ -\sum\limits_{ip\in\Psi_i} L_{ip,k}M_{ip}; & k \neq i, \end{cases} \quad (3)$$

where:

- M_i power generation at the bus i
- $-M_{ip}$ power flow into the bus i from the bus p
- $L_{ip,k}$ contribution of the generator k to the power flow on the line ip
- Ψ_i a set consisting of all lines which supply the bus i, i.e. the lines for which $M_{ip} < 0$.

The share of every generator supplying the bus i is as follows:

$$K_{i,k} = \frac{M_{i,k}}{\sum\limits_{k\in\Gamma} M_{i,k}}, \quad (4)$$

where Γ represents a set of all generators in the power system.

Determination of generator's contributions to the customer load requires a power-mixing rule at a bus, Fig. 1. For that purpose, the proportional sharing principle is used, which has been argued with the game theory [2], [8]. It assumes a linear relation of the power outflow from a bus to the sum of the power inflows. The principle also applies to the loads at that bus.

Let the bus i supply the loads s and lines j as shown in Fig. 1. The observed load M_{ds}, can be expressed in terms of the power inflows:

$$M_{ds} = -\alpha_{ds} \sum_{ip\in\Psi_i} M_{ip}, \quad (5)$$

where the coefficient α_{ds} is a linear factor.

The generators' share in the customer load $(\beta_{ds,k})$ is calculated using the proportional sharing principle. This way, the share of each generator supplying the bus i is equal to the generators' share in the load s:

$$\beta_{ds,k} = K_{i,k}. \quad (6)$$

Usually, it is more interesting to know the amount of the active or reactive power supplied by the particular generator to a customer load. That can be easily calculated taking into account (6):

$$M_{ds,k} = \beta_{ds,k}M_{ds}, \quad (7)$$

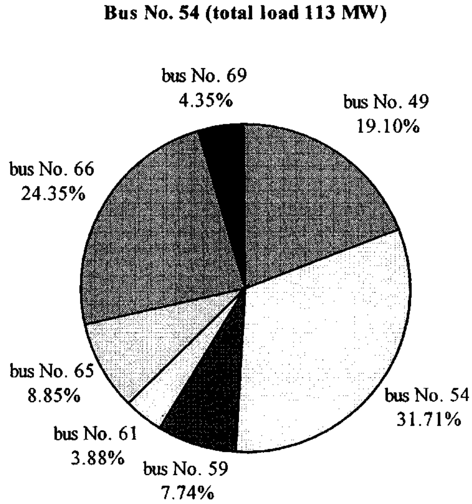

Fig. 2. Generators' contributions to the bus no. 54 active power load demand.

where: $M_{ds,k}$—power supplied by the generator k to the customer s, M_{ds}—entire consumption of the customer s.

The generators' contribution factors in the customer load take values between zero and one:

$$0 \leq \beta_{ds,k} \leq 1. \quad (8)$$

For all generators in the power system, the sum of the factors equals to:

$$\sum_{k\in\Gamma} \beta_{ds,k} = 1. \quad (9)$$

III. Active Power Results

The method for calculating generators' shares in a customer load was extensively tested. Some results are presented for the IEEE118 test system and for the Slovenian power system. Before calculation, the line power flows have to be determined using either power flow calculation or results obtained from the state estimator.

A. IEEE118 Test System

In Fig. 2, a base case of the generator's distribution in a load supply for the bus no. 54 is presented. It can be seen that the nearest generators, buses Nos. 54, 66 and 49, supply a great part of this load, and that in total it is supplied by seven generators.

Let a customer supplied by the bus no. 54 purchase additional 100 MW from the near-by generator at the bus no. 46. The result is given in Fig. 3, where only a negligible portion of additional power is received from the generator no. 46, since the other generators in the vicinity increased their share sufficiently.

Table I gives an overview of generators supplying bus loads before and after the purchase. The underlined numbers denote the changes in the supply pattern. Due to that purchase, the generator no. 46 started supplying many new loads, similarly as the-generators Nos. 69 and 80. The purchase affects the supply pattern of almost one third of the load buses.

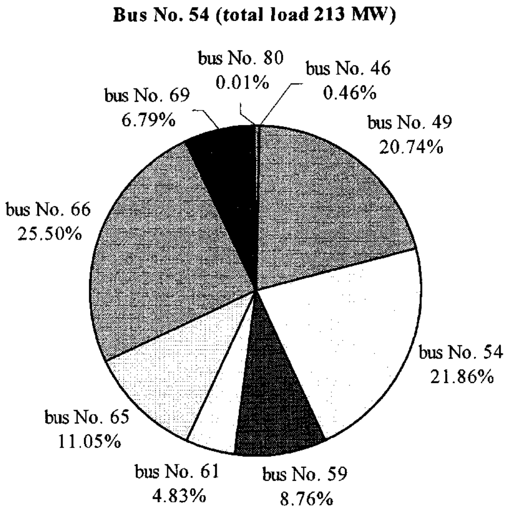

Fig. 3. Generators' contributions to the bus no. 54 active power demand after a customer at this bus purchase additional 100 MW from the power plant at the bus no. 46.

B. IEEE118 Test System, Influence of Purchase on Other Customers

It is also interesting to observe the effect of a purchase of electric power on the supply of customers, which do not take part in a particular transaction. As an example, the base case for the bus no. 47 is presented in Fig. 4. The change in its supply pattern after a customer at the bus no. 54 has purchased additional 100 MW from the generator at the bus no. 46, is shown in Fig. 5.

Initially, four generators supplied the bus no. 47 with the active power, where the bus no. 69 contribution amounted to more than 86%. It can be seen that the supplying share of the bus no. 47 has changed significantly after the customer at the bus no. 54 purchased additional active power from the generator at the bus no. 46. Three generators stopped to feed the bus No. 47 andthe generator at the bus no. 46 completely took their place, although its power was purchased for the no. 54 load.

This leads to a conclusion important for operation of the deregulated environment. The tracing of purchased power is possible offering detailed analyzes of actual power flows compared to those foreseen by contracts. This fact may influence the transmission service costing methods as well as the congestion management methods.

C. Slovenian Power System

The Slovenian power system, containing 400 kV, 220 kV, and 110 kV networks, modeled with the boundary buses consists of 275 buses, 48 power plants and 448 lines. The peak load is 1750 MW.

The contributions of the power plants to the supply at the Ljubljana-center bus are shown in Fig. 6. The total consumption at this bus is 38.7 MW. Customers connected to the Ljubljana-center bus are supplied by three power plants, which are geographically located nearby. The situation changed if a customer connected to the Ljubljana-center bus decided to buy

TABLE I
SUPPLY PATTERN OF LOADS BY THE GENERATORS BEFORE AND AFTER PURCHASE

Gen. at the bus No.	Supplying load at the bus No.	
	Before 100 MW purchase	After 100 MW purchase
10	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 113, 117, 119, 120, 121	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 113, 117, 119, 120, 121
12	1, 2, 3, 12, 14, 15, 16, 19, 33, 117	1, 2, 3, 12, 14, 15, 16, 19, 33, 117
25	19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 31, 32, 72, 113, 114, 115, 124	19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 31, 32, 72, 113, 114, 115, 124
26	15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 72, 113, 114, 115, 121, 123, 124	15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 72, 113, 114, 115, 121, 123, 124
31	29, 31	29, 31
46	43, 44, 45, 46, 127	19, 34, 36, 40, 41, 42, 43, 44 45, 46, 47, 48, 49, 50, 51, 52 53, **54**, 55, 56, 57, 58, 126, 128
49	40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, **54**, 55, 56, 57, 58, 126, 127, 128	19, 34, 36, 40, 41, 42, 43, 44 45, 48, 49, 50, 51, 52, 53, **54** 55, 56, 57, 58, 126, 128
54	53, **54**, 55, 56	53, **54**, 55
59	53, **54**, 55, 56, 59	53, **54**, 55, 56, 59, 129
61	53, **54**, 55, 56, 59, 60, 61, 62	53, **54**, 55, 56, 59, 60, 61, 62 129
65	19, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, **54**, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 116, 125, 126, 127, 128, 129, 130	19, 33, 34, 35, 36, 37, 38, 39 40, 41, 42, 43, 44, 45, 48, 49 50, 51, 52, 53, **54**, 55, 56, 57 58, 59, 60, 61, 62, 63, 64, 65 66, 67, 125, 126, 127, 128, 129
66	40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, **54**, 55, 56, 57, 58, 59, 60, 62, 66, 67, 126, 127, 128, 129	19, 34, 36, 40, 41, 42, 43, 44 45, 48, 49, 50, 51, 52, 53, **54** 55, 56, 57, 58, 59, 60, 62, 66 67, 126, 127, 128, 129
69	24, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, **54**, 55, 56, 57, 58, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 116, 118, 122, 126, 127, 128, 132	19, 24, 33, 34, 35, 36, 37, 38 39, 40, 41, 42, 43, 44, 45, 47 48, 49, 50, 51, 52, 53, **54**, 55 56, 57, 58, 59, 60, 61, 62, 63 64, 65, 66, 67, 68, 69, 70, 71 72, 73, 74, 75, 76, 77, 78, 116 118, 122, 125, 126, 127, 128 129, 130, 132
80	24, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 95, 96, 97, 98, 99, 116, 118, 122, 131, 132	19, 33, 34, 35, 36, 37, 38, 39 40, 41, 42, 43, 44, 45, 48, 49 50, 51, 52, 53, **54**, 55, 56, 57 58, 59, 60, 61, 62, 63, 64, 65 66, 67, 68, 74, 75, 76, 77, 78 79, 80, 81, 82, 95, 96, 97, 98 99, 116, 118, 125, 126, 127 128, 129, 130, 131
87	86, 87	86, 87
89	24, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 118, 122, 132, 133, 134	74, 75, 76, 77, 78, 82, 83, 84 85, 86, 88, 89, 90, 91, 92, 93 94, 95, 96, 98, 99, 100, 101 102, 103, 104, 105, 106, 107 108, 109, 110, 112, 118, 133 134
100	98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 112, 134	98, 99, 100, 103, 104, 105, 106 107, 108, 109, 110, 112, 134
103	103, 104, 105, 106, 107, 108, 109, 110, 112, 134	103, 104, 105, 106, 107, 108 109, 110, 112, 134
111	110, 111, 112	110, 111, 112

additional 40 MW from the remote Krško Nuclear Power plant (NPP), shown in Fig. 7.

The contribution of the Krško NPP to the load at the Ljubljana-center bus is instead of 40 MW only 0.75 MW, what equals to 0.95% of the new load. Moreover, the number of power plants supplying load at the Ljubljana-center bus has

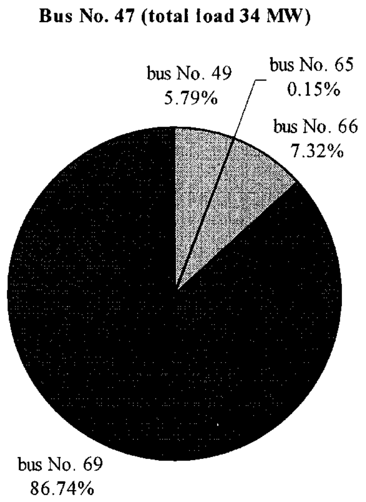

Fig. 4. Generators' contributions to the bus no. 47 active power demand, the base case.

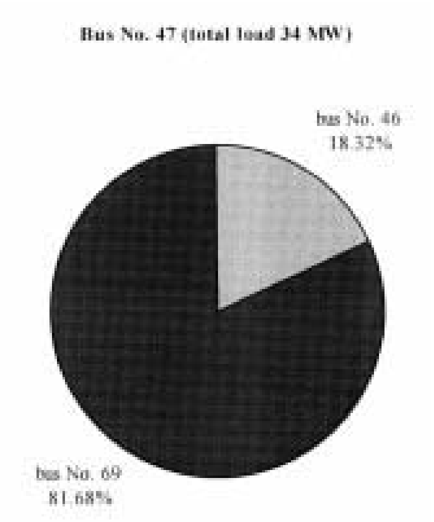

Fig. 5. Generators' contributions to the bus no. 47 active power demand after a customer at the bus no. 54 purchase additional 100 MW from the power plant at the bus no. 46.

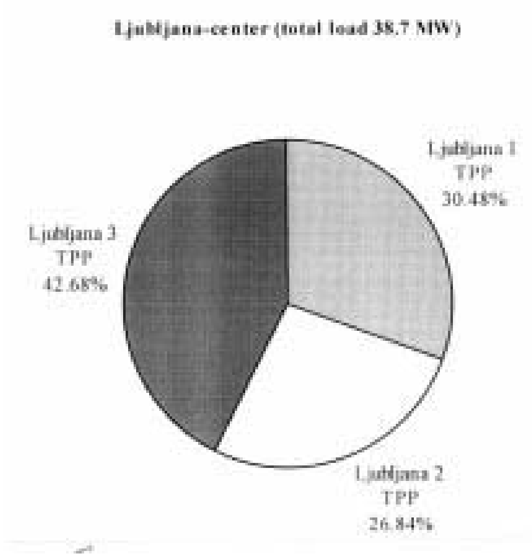

Fig. 6. Generators' contributions in supplying customers at the Ljubljana center bus.

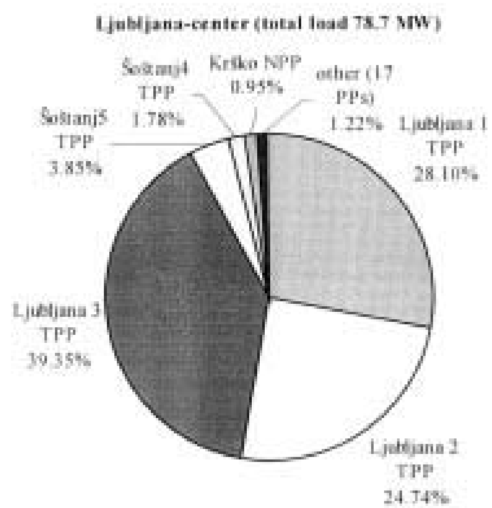

Fig. 7. Generators' contributions in supplying customers at the Ljubljana-center bus after purchase of additional 40 MW at the Krško NPP.

increased from 3 to 23. Contributions of power plants located near this bus have been mostly increased. Beside that, the transmission losses in the whole system increased by 1.54 MW.

Essentially the same situation as in the previous case is shown in Fig. 8, with the only difference that additional 40 MW is now purchased from the Ljubljana 2 Thermal Power plant (TPP) located near the Ljubljana-center bus.

After this purchase, only three power plants supply customers at the Ljubljana-center bus. The main difference from the base case is an increased contribution of the Ljubljana 2 TPP. The transmission losses increased by 0.20 MW compared to the base case. This amounts to 1.34 MW less than in the case when the additional power is purchased from the Krško NPP. The tests on the Slovenian power system confirmed the results obtained for the IEEE118 test system.

IV. Reactive Power Results

The same situation as in the Section III-A for the IEEE118 test system is presented in Figs. 9 and 10 for reactive power. The results for the base case show that the bus no. 54 is mostly supplied with reactive power by the bus no. 49. In total, four generators supply this bus with reactive power.

The results indicate that buses are usually supplied from the nearest reactive power generator buses. As in the previous example, the customer at the bus no. 54 purchases additional 100 MW from the power plant at the bus no. 46. From Fig. 10, it can be observed that the bus no. 49 contribution in supplying the bus no. 54 with reactive power has been significantly decreased while contribution of the bus no. 54 has been increased to a great extent.

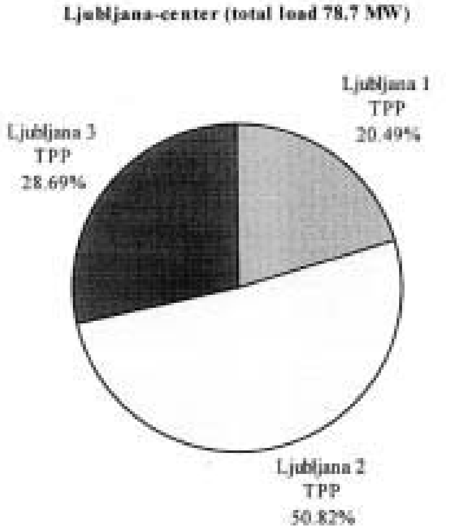

Fig. 8. Generators' contributions in supplying load at the Ljubljana-center bus after purchase of additional 40 MW at the Ljubljana 2 TPP.

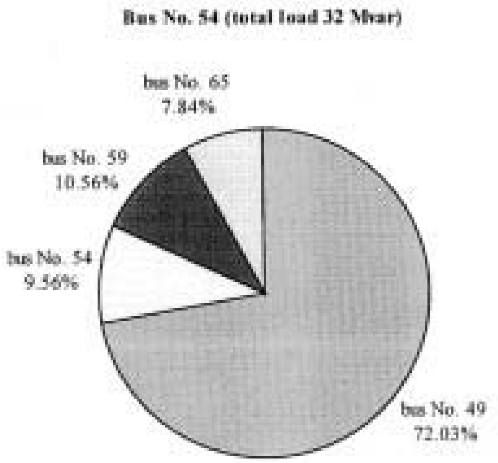

Fig. 9. Generators' contributions to the bus no. 54 reactive power demand.

V. Conclusion

In the paper, a straightforward method for determining the share of generators in a customer load has been developed. It is based on the nodal generation distribution factors, which do not use the system matrices. With appropriate use of these factors, the generators' contributions to a customer load are obtained for the active and reactive powers. The method uses either the measured line flow values or those obtained by the power flow calculation.

The two different power systems results demonstrate that the electric power flows according to the physical rather than economical rules imposed by power systems deregulation. The method of the NGDF-s could be exploited for transmission service pricing and congestion management since the factors clearly yield the exact active and reactive power share of aparticipant in the line loading. Therefore, the factors could be used in the reactive power management for determination of the generator influence area, as well.

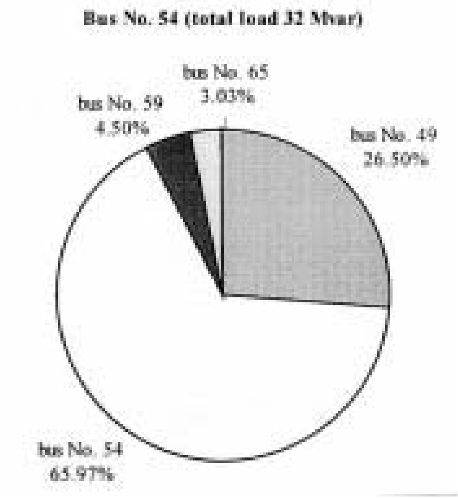

Fig. 10. Generators' contributions to the bus no. 54 reactive power demand after a customer at this bus purchase additional 100 MW from the power plant at the bus no. 46.

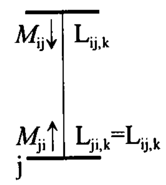

Fig. 11. Conservation of the contribution factors along the line.

Appendix
The Nodal Generation Distribution Factors

The nodal method for determining the share of every generator in the particular line power flow is based on a search algorithm. It searches for power flow directions and the matrix calculations are not required. It yields the nodal generation distribution factors (NGDF-s), which represent the share of every generator in a particular line power flow. Contrary to the other methods [1], [2], [5], the proposed method handles besides active also reactive powers, since the factors do not change along the line as shown in Fig. 11. Moreover, the line losses could be simply determined by NGDF-s.

The NGDF-s are calculated for lines connected to the power source buses first. Not every generator bus is a source bus. The power source buses are defined as buses where all injected powers to the connected lines are positive. If the bus r is a source bus, then for all connected lines with an adjacent bus j, the following applies:

$$M_{rj} \geq 0. \tag{10}$$

Since all lines connected to the source bus are supplied only by this bus, the NGDF-s for these lines equals to 1 for generator, which supplies these lines, and equals to 0 for all other generators:

$$L_{rj,k} = \begin{cases} 1; & k = r \\ 0; & k \neq r, \end{cases} \tag{11}$$

TABLE II
NGDF VS. TGDF COMPARISON

	Generator 1		Generator 2	
Line	TGDF	NGDF	TGDF	NGDF
1-2	1.000	1.000	0.000	0.000
1-3	1.000	1.000	0.000	0.000
1-4	1.000	1.000	0.000	0.000
2-4	0.345	0.341	0.655	0.659
4-3	0.605	0.602	0.395	0.398

where $L_{rj,k}$ stands for a NGDF of the line rj and for the generator at the bus k. Following this rule, the NGDF-s for all lines supplied by the source buses could be determined.

The NGDF-s for all lines, which are not directly supplied by the source buses, are calculated next. Let Ψ_i denotes a set of all lines supplying the bus i and Ξ_i a set of lines supplied by the bus i. It is important that $L_{ip,k}$ for all lines from the set $ip \in \Psi_i$ are calculated in advance. For the bus i, it can be calculated, to what degree it is supplied by every particular generator in the system, applying (3).

Then, the proportional sharing principle is used. The practical result of this principle is, that the NGDF-s of all lines fed by the bus i are equal to:

$$L_{ij,k} = \frac{M_{i,k}}{\sum_{k \in \Gamma} M_{i,k}} \qquad \text{for all } ij \in \Xi_i, \tag{12}$$

where Γ denotes a set of all generator buses in the system.

The described calculation has to be done for all buses in the power system yielding the NGDF-s for all lines. These factors can be interpreted as a share of each generator in the power flow on a selected line.

The factors have been compared to the matrix based TGDF-s. The comparison of results for the test system [2] shows the differences below 1%, Table II.

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