

Ekpyrotic and Cyclic Cosmology

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Abstract

Ekpyrotic and cyclic cosmologies provide theories of the very early and of the very late universe. In these models, the big bang is described as a collision of branes - and thus the big bang is not the beginning of time. Before the big bang, there is an ekpyrotic phase with equation of state $w = \frac{\mathcal{P}}{\rho} \gg 1$ (where \mathcal{P} is the average pressure and ρ the average energy density) during which the universe slowly contracts. This phase resolves the standard cosmological puzzles and generates a nearly scale-invariant spectrum of cosmological perturbations containing a significant non-gaussian component. At the same time it produces small-amplitude gravitational waves with a blue spectrum. The dark energy dominating the present-day cosmological evolution is reinterpreted as a small attractive force between our brane and a parallel one. This force eventually induces a new ekpyrotic phase and a new brane collision, leading to the idea of a cyclic universe. This review discusses the detailed properties of these models, their embedding in M-theory and their viability, with an emphasis on open issues and observational signatures.

Key words: cosmology, branes, big bang, dark energy

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1 Introduction

The evolution of our universe is very well understood from the time of big bang nucleosynthesis until the present. However, if we go beyond these time frontiers, almost nothing is known with a great amount of certainty. We know that the early universe must have been in a very special state (very homogeneous and flat, but with tiny fluctuations in curvature), but we do not know why. The theory of inflation [41,69,4] provides a possible explanation by means of a period of rapid expansion preceding nucleosynthesis, but it has not yet been possible to test inflation conclusively, which is why it is important to keep an open mind. Also, we know that dark energy has come to dominate the energy density of the universe [86,85] and that it will determine the expansion of the universe in the near future. If dark energy turns out to be a cosmological constant, which is the theoretically simplest way of modeling it, then the universe will continue to expand at an accelerating pace and become increasingly dilute and cold, making life very difficult, if not impossible. However, it should be borne in mind that we have no fully convincing explanation of dark energy at present.

This review will be concerned with a set of ideas, strongly inspired by string theory, that suggests alternative solutions to the early universe puzzles mentioned above, and considers an alternative fate for the future of our universe. What makes these ideas exciting is that, on the one hand, they provide a theoretical playground for applying string theory to cosmology, and, on the other hand, certain observational signatures predicted from these models are in a range that will be tested by near-future satellite experiments. This provides a certain timeliness to a subject concerned with eras far removed from everyday experience.

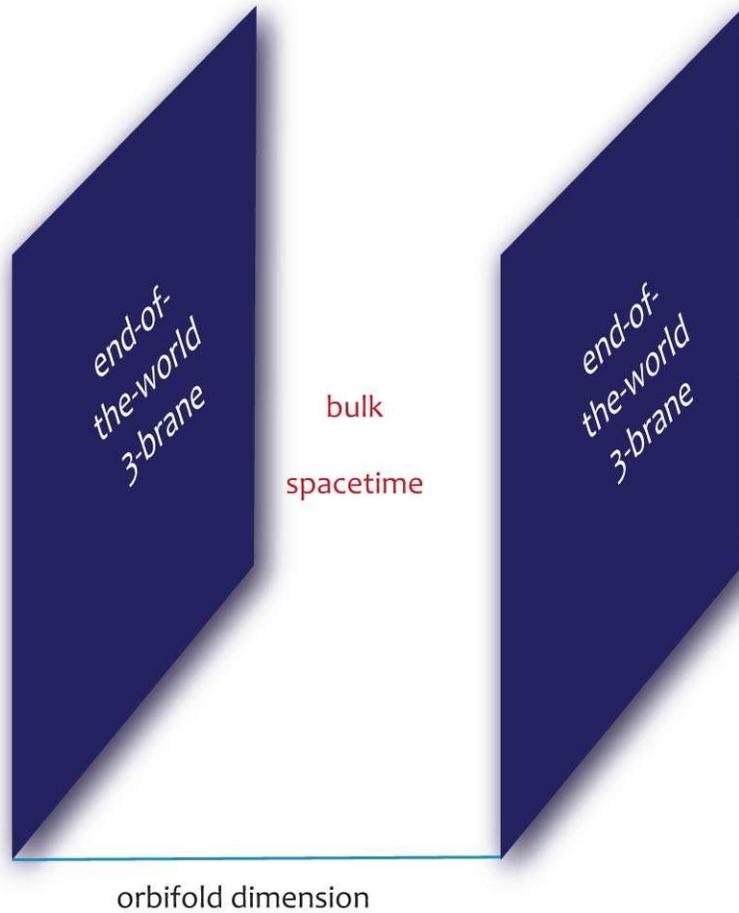


Fig. 1. The braneworld picture of our universe. Think of a sandwich: the filling is the 5-dimensional bulk spacetime, which is bounded by the two pieces of bread a.k.a. the 4-dimensional boundary branes. There is no space “outside” of the sandwich, but the branes can be infinite in all directions perpendicular to the line segment. In the M-theory embedding, there are 6 additional internal dimensions at each 5-dimensional spacetime point.

Ekpyrotic¹ and cyclic cosmology are based on the braneworld picture of the universe, in which spacetime is effectively 5-dimensional, but with one dimension not extending indefinitely, but being a line segment², see Fig. 1. The endpoints of this line segment (orbifold) are two (3+1)-dimensional boundary branes. All matter and forces, except for gravity, are localized on the branes, while gravity can propagate in the whole spacetime. Our universe, as we see it, is identified with one of the boundary branes and, as long as the branes are far apart, can interact with the other brane only via gravity. The ekpyrotic model

¹ The name ekpyrosis can be translated as all-engulfing cosmic fire. In Stoic philosophy, it represents the contractive phase of eternally-recurring destruction and re-creation [1].

² This setting will be discussed in detail in section 6, along with its motivation from and embedding in heterotic M-theory.

assumes that there is an attractive force between the two branes, which causes the branes to approach each other. This ekpyrotic phase has the rather non-intuitive property that it flattens the branes to a very high degree. Eventually the two branes collide and move through each other (since there is no space “outside” of the boundary branes, it makes no difference whether we say that the branes bounce off each other or move through each other). It is this collision that, from the point of view of someone living on one of the branes, looks like the big bang. The collision is slightly inelastic and produces matter and radiation on the branes, where the standard cosmological evolution now takes place. However, due to quantum fluctuations, the branes are slightly rippled and do not collide everywhere at exactly the same time. In some places, the branes collide slightly earlier, which means that the universe has a little bit more time to expand and cool. In other places, the collision takes place slightly later, and those regions remain a little hotter. This provides a heuristic picture of the way temperature fluctuations are naturally produced within the model. Shortly after the brane collision, the distance between the boundary branes gets almost stabilized, but the branes start attracting each other again very slightly. This very slight attraction acts as quintessence, and is identified with the dark energy observed in the universe. After a long time and as the branes become closer again, they start attracting each other more strongly so that we get a new ekpyrotic phase and eventually a new brane collision with the creation of new matter. In this way, a cyclic model of the universe emerges.

The preceding description of course only provided a very rough outline of the main ideas. All of this will be made precise in the following. However, at this stage it is already clear that conceptually the cyclic universe differs substantially from the standard big bang picture. For one, the big bang is seen as a physical event and not a mysterious moment of creation. As such, it does not represent the beginning of time (note that if quantum gravity is unitary, this point of view seems unavoidable). Thus there was plenty of time before the big bang for the universe to be in causal contact over large regions, and in this way the horizon problem is automatically solved. The long timescales existing before the big bang also imply that other cosmological puzzles don't have to be solved in the extremely short time interval between the big bang and nucleosynthesis. What's more, properties of the very early universe (such as cosmological perturbations) and properties of the late universe (such as dark energy) are described by the same ingredients, namely the motion of branes.

In the description above, the higher-dimensional interpretation was emphasized. This higher-dimensional viewpoint is useful for understanding the origin of the ideas, and provides a concrete realization of them. However, it is important to realize that many of the processes mentioned above can also be discussed purely in a 4-dimensional effective theory. Some processes, such as the ekpyrotic period of slow contraction before the big bang, are quite gen-

eral and could have a different origin than the brane motion just described. Therefore, the four subsequent sections of this review, which describe the fundamentals of ekpyrotic and cyclic cosmology, are phrased mostly in terms of a 4-dimensional effective theory. Section 2 describes the ekpyrotic phase and the way it resolves the standard cosmological puzzles. Section 3 describes the approach to the big crunch in more detail. The cosmological fluctuations generated by the model are discussed in section 4. This includes a treatment of scalar perturbations with their second-order non-gaussian corrections as well as tensor perturbations. The following section is concerned with the details of the cyclic universe, such as its relationship to the second law of thermodynamics and its global structure. Section 6 explains the origin of the 4-dimensional effective theory used in the preceding sections and its relation to string and M-theory. Finally, section 7 provides a brief conclusion and an outlook to possible future developments.

It should be noted from the outset that ekpyrotic and cyclic cosmology are theories that are still in full development. Certain aspects of the theories have gained or lost in importance over time. Here are two examples: in the original ekpyrotic proposal, the brane collision was between a boundary and a bulk brane, the second boundary brane being a spectator. It was later realized that the model could be simplified by considering a collision of the boundary branes. It was also thought that the dark energy phase was crucial for smoothing and isotropizing the universe. Again, it was later realized that the ekpyrotic phase can achieve this on its own. In this review the subject will be discussed only according to its present understanding. Moreover, every effort will be made to highlight assumptions that are in need of a precise calculation, or that are due for example to limitations in our understanding of string theory. This should hopefully provide the reader with opportunities to contribute to the subject.

2 The Ekpyrotic Phase

The ekpyrotic phase is the central insight that shows how a contracting phase preceding the big bang can solve the standard cosmological puzzles [48,35]. This is a surprising statement since one would naively think that a contracting, gravitating system would lead to large curvatures near a singularity. And we know from the near-flatness of our current universe that shortly after the big bang the universe must have been extremely flat. This is usually referred to as the flatness problem. Let us briefly quantify this problem; at the same time this will serve to set up notation. Consider a Friedmann-Robertson-Walker

(FRW) metric³

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega_2^2 \right) \quad (1)$$

where $a(t)$ denotes the scale factor of the universe and $\kappa = -1, 0, 1$ for an open, flat or closed universe respectively. If we consider the universe to be filled with a perfect fluid (which is a good approximation for many types of matter), with energy-momentum tensor

$$T_{\nu}^{\mu} = \text{diag}[-\rho, \mathcal{P}, \mathcal{P}, \mathcal{P}], \quad (2)$$

where ρ denotes the energy density and \mathcal{P} the pressure, then the Einstein equations reduce to the so-called Friedmann equations

$$H^2 = \frac{1}{3}\rho - \frac{\kappa}{a^2} \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3\mathcal{P}), \quad (4)$$

where the Hubble parameter is defined by $H \equiv \frac{\dot{a}}{a}$ and a dot denotes a derivative w.r.t. time t . The Bianchi identity on the Einstein tensor $\nabla^{\nu} G_{\mu\nu} = 0$ requires for consistency that the energy-momentum tensor be covariantly conserved, *i.e.* $\nabla^{\nu} T_{\mu\nu} = 0$, which leads to the continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \mathcal{P}) = 0. \quad (5)$$

For a fluid with a constant equation of state $w \equiv \frac{\mathcal{P}}{\rho}$, the continuity equation can be integrated to give

$$\rho \propto a^{-3(1+w)}. \quad (6)$$

We will use this scaling relation on many occasions. Now define the quantity $\Omega(t) \equiv \frac{\rho(t)}{\rho_{crit}(t)}$ where $\rho_{crit} = 3H^2$ denotes the critical density. Then the first Friedmann equation (3) can be rewritten as

$$\Omega - 1 = \frac{\kappa}{(aH)^2} \quad (7)$$

and it expresses how close the universe is to flatness. At the present time, observations show that [52]

$$|\Omega - 1|_0 \lesssim 10^{-2}. \quad (8)$$

Extrapolating back in time, this means that at the Planck time

$$\frac{|\Omega - 1|_{Pl}}{|\Omega - 1|_0} = \frac{(aH)_0^2}{(aH)_{Pl}^2}. \quad (9)$$

³ I will mostly use natural units $\hbar = c = 1$ and $8\pi G = M_{Pl}^{-2} = 1$.

If we assume a radiation-dominated universe ($w = 1/3$), which is a good approximation for the present calculation, then the continuity and Friedmann equations lead to $\rho \propto a^{-4}$ and $a \propto t^{1/2}$, so that

$$|\Omega - 1|_{Pl} \lesssim \frac{t_{Pl}}{t_0} 10^{-2} \sim 10^{-62}. \quad (10)$$

Even though extrapolating all the way back to the Planck time is probably exaggerated, this simple estimate shows that the universe must have been extremely flat at early times. Clearly, this peculiar observation asks for an explanation.

In order to understand how the ekpyrotic phase can resolve this problem, it is useful to briefly review the solution proposed in inflationary theory. Inflation postulates a period of rapid expansion immediately following the big bang, during which the equation of state $w = \frac{\mathcal{P}}{\rho}$ of the universe is close to -1 [41,69,4]. From the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3\mathcal{P}) = -\frac{\rho}{6}(1 + 3w), \quad (11)$$

we can see that such an equation of state indeed leads to accelerated expansion. One way to model a matter component with the required equation of state is to have a scalar field ϕ with canonical kinetic energy and with a very flat potential $V(\phi)$, *i.e.* we add the following terms to the Lagrangian

$$\sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right). \quad (12)$$

Then a quick calculation shows that (in the absence of spatial gradients) the equation of state is given by

$$w_\phi = \frac{\mathcal{P}_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}, \quad (13)$$

which is close to -1 if the potential is sufficiently flat so that the fields are rolling very slowly, *i.e.* if $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$. In the presence of different matter types, represented here by their energy densities ρ , the Friedmann equation (3) generalizes to

$$H^2 = \frac{1}{3} \left(\frac{-3\kappa}{a^2} + \frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\sigma^2}{a^6} + \dots + \frac{\rho_\phi}{a^{3(1+w_\phi)}} \right) \quad (14)$$

The subscript m refers to non-relativistic matter and includes dark matter, r refers to radiation and σ denotes the energy density of anisotropies in the curvature of the universe (the associated scaling with a will be calculated in the following section). In an expanding universe, as the scale factor a grows, matter components with a slower fall-off of their energy density come to dominate.

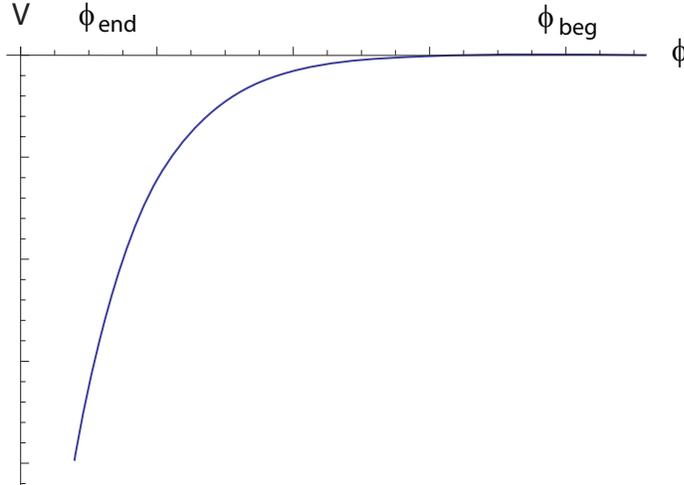


Fig. 2. The potential during ekpyrosis is negative and steeply falling; it can be modeled by the exponential form $V(\phi) = -V_0 e^{-c\phi}$.

Eventually, the inflaton, whose energy density is roughly constant, dominates the cosmic evolution and determines the (roughly constant) Hubble parameter while causing the scale factor to grow exponentially,

$$a \propto e^{Ht}. \quad (15)$$

We can define the relative energy density in the curvature as $\Omega_\kappa \equiv -\kappa/(a^2 H^2)$ and in the anisotropies as $\Omega_\sigma \equiv \sigma^2/(3a^6 H^2)$. During inflation, these relative densities fall off quickly, and the universe is rendered exponentially flat; according to (10) the flatness puzzle is then resolved as long as the scale factor grows by at least 60 e-folds.

Now we will show how the same problem can be solved by having a contracting phase before the standard expanding phase of the universe. The Friedmann equation relates the Hubble parameter to the total energy density in the universe, which is the sum of kinetic and potential energy. Now suppose that, instead of a flat potential, the scalar ϕ has a very steep, negative potential, as shown in Fig. 2. As a concrete example, one can model the potential with a negative exponential

$$V(\phi) = -V_0 e^{-c\phi}, \quad (16)$$

where V_0 and c are constants. Let us give one brief motivation for a negative potential: foreshadowing a cyclic picture, we can ask if it is possible for the universe to revert from an expanding to a contracting phase in a non-singular way. Then at some point the Hubble parameter must go through zero, which can be achieved by having a negative potential in order to cancel the positive kinetic energy of matter. From (13) this automatically implies an equation of state $w > 1$, and will be seen to have very powerful consequences. In a contracting universe, the argument presented in the previous paragraph is reversed, and one would initially expect the anisotropy term (proportional to

a^{-6}) to come to dominate the cosmic evolution. However, if there is a matter component with $w > 1$, then (6) implies that this component will scale with an even larger negative power of a , and hence will come to dominate the cosmic evolution in a contracting universe in the same way as the inflaton comes to dominate in an expanding universe.

In fact it is straightforward to generalize the treatment to having many scalars ϕ_i with potentials $V_i(\phi_i)$. Then, in a flat FRW background, the equations of motion become

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{i,\phi_i} = 0 \quad (17)$$

and

$$H^2 = \frac{1}{3} \left[\frac{1}{2} \sum_i \dot{\phi}_i^2 + \sum_i V_i(\phi_i) \right], \quad (18)$$

where $V_{i,\phi_i} = (\partial V_i / \partial \phi_i)$ with no summation implied. A useful relation is

$$\dot{H} = -\frac{1}{2} \sum_i \dot{\phi}_i^2. \quad (19)$$

If all the fields have negative exponential potentials $V_i(\phi_i) = -V_i e^{-c_i \phi_i}$ and if $c_i \gg 1$ for all i , then the Einstein-scalar equations admit the scaling solution

$$a = (-t)^p, \quad \phi_i = \frac{2}{c_i} \ln(-\sqrt{c_i^2 V_i / 2t}), \quad p = \sum_i \frac{2}{c_i^2}. \quad (20)$$

Thus, we have a very slowly contracting universe with (constant) equation of state

$$w = \frac{2}{3p} - 1 \gg 1. \quad (21)$$

We are using a coordinate system in which the big crunch occurs at $t = 0$; in other words, the time coordinate is negative during the ekpyrotic phase. The steeply falling scalar fields act as a very stiff fluid, and, in fact, one can take the condition $w \gg 1$ (or equivalently $p \ll \frac{1}{3}$) to be the defining feature of ekpyrosis. The matter content does not necessarily have to be composed of scalar fields, but it is easy to model the ekpyrotic phase that way, and scalar fields commonly appear in effective theories arising from higher dimensions. The main consequence is that the extra term in the Friedmann equation (14) with $w \gg 1$ comes to dominate the cosmic evolution, and once more, the fractional energy densities $\Omega_\kappa \propto a^{-2} H^{-2}$ and $\Omega_\sigma \propto a^{-6} H^{-2}$ quickly decay. Thus, neglecting quantum effects, the universe is left exponentially flat and isotropic as it approaches the big crunch. As we will see in section 4, the inclusion of quantum effects superposes small fluctuations on this classical background.

What are the requirements on the ekpyrotic phase for the flatness problem to be solved? First, we must ensure that $w > 1$, which means that the potentials have to be steep enough. In the single-field case for example, from (21) we see

that we need $c_1 > \sqrt{6}$. Also, from the scaling solution (20), we see that the scale factor a remains almost constant, while the Hubble parameter $H \propto t^{-1}$ during ekpyrosis. In order to solve the flatness problem, aH has to grow by at least 60 e-folds in magnitude, see (7) and (10), which means that

$$|t_{beg}| \geq e^{60} |t_{end}|, \quad (22)$$

where the subscripts *beg* and *end* refer to the beginning and the end of the ekpyrotic phase respectively. As will be discussed in section 4, we need $t_{end} \approx -10^3 M_{Pl}^{-1}$ in order to obtain the observed amplitude of cosmological perturbations, so that

$$|t_{beg}| \geq 10^{30} M_{Pl}^{-1} = 10^{-13} s. \quad (23)$$

This is the minimum time the ekpyrotic phase has to last in order to solve the flatness problem, under the assumption that a flat universe undergoing a crunch/bang transition can reemerge as a flat universe afterwards. We will discuss this assumption repeatedly in what follows.

In the cyclic picture of the universe, the potential V interpolates between the GUT scale and the dark energy scale. From the scaling solution (20), we have that $V \propto t^{-2}$, and this leads to

$$|t_{beg}| = \sqrt{\frac{V_{end}}{V_{beg}}} |t_{end}| \approx \sqrt{10^{112}} 10^3 M_{Pl}^{-1} = 10^{16} s. \quad (24)$$

In this case, the ekpyrotic phase lasts on the order of a billion years, and it easily resolves the flatness puzzle.

From the higher-dimensional point of view, one of the scalar fields ϕ_i is the radion, which determines the distance between the branes (the higher-dimensional picture will be discussed in detail in section 6). The potential then represents a conjectured attractive force between the end-of-the-world branes. By diluting inhomogeneities in the brane curvature, the ekpyrotic phase causes the branes to become very flat and parallel over large patches. Thus the homogeneity puzzle, namely the question of how different parts of the sky can have approximately the same temperature despite the fact that they could not have been in causal contact since the big bang, is also solved by the ekpyrotic phase.

Moreover, as will be discussed in sections 3 and 6, the brane scale factors remain finite even at the collision. Semi-classical calculations confirm the intuition that the collision should be slightly inelastic, so that matter is produced at a finite temperature [104]. And as long as this temperature is below the GUT scale (which might depend on the collision rapidity being low enough), topological defects will not form, and thus the monopole problem is avoided. In

this way all the standard cosmological puzzles can be resolved by the ekpyrotic phase.

3 Towards the Crunch

We have just seen how the simple assumption of a contracting phase with equation of state $w > 1$, preceding the usual expanding phase of the universe, manages to solve the flatness and homogeneity puzzles of standard big bang cosmology. But now we will see that it does much more than that: it provides a way of avoiding Belinsky-Khalatnikov-Lifshitz (BKL) chaotic mixmaster behavior [12], it prepares the universe in the most favorable state as it approaches the big crunch and, on top of that, it generates a nearly scale-invariant spectrum of scalar perturbations that act as the seeds for the large-scale structure of the universe.

3.1 Avoiding Chaos

Any cosmological theory involving a contracting phase must address the issue of chaos: from the studies of BKL [12], it is known that if all matter components have an equation of state $w < 1$, then a contracting universe is unstable w.r.t small perturbations. The metric becomes highly anisotropic, and of Kasner form (see below). Typically all spatial dimensions except for one shrink away, but repeatedly the metric jumps from one Kasner form to another one, thus leading to BKL oscillations or “chaotic mixmaster” behavior [79]. In this case all predictability in going towards a big crunch seems to be lost.

In order to better understand the mechanism of the ekpyrotic phase, and to address both the issues of chaos and cosmological perturbations, it is useful to look in more detail at the evolution of anisotropies. In synchronous gauge, we can write the metric as

$$ds^2 = -dt^2 + g_{ij}(x^\mu)dx^i dx^j. \quad (25)$$

In this gauge, we can choose a time coordinate such that the big crunch occurs everywhere at $t = 0$. From the work of BKL [12] we know that in a contracting universe spatial gradients quickly become irrelevant compared to time gradients, so that we can consider the simplified metric

$$ds^2 = -dt^2 + a(t)^2 \sum_i e^{2\beta_i(t)} dx^i dx^i, \quad (26)$$

which is of Kasner form and where we require $\sum_i \beta_i = 0$. In other words,

the dynamics becomes ultralocal and the Einstein equations reduce to the Friedmann equations

$$3H^2 = \frac{1}{2} \sum_i \dot{\beta}_i^2 + \dots \quad (27)$$

$$\ddot{\beta}_i + 3H\dot{\beta}_i = 0. \quad (28)$$

Then the growing-mode solution to the last equation is $\dot{\beta}_i \propto a^{-3}$, which means that the first Friedmann equation can be rewritten as

$$3H^2 = \frac{\sigma^2}{a^6} + \dots \quad (29)$$

for a constant σ parameterizing the anisotropic energy density at unit scale factor. Hence, as stated in the previous section, anisotropies blueshift proportionally to a^{-6} and would quickly dominate the cosmological evolution if it wasn't for the ekpyrotic scalar whose energy density grows faster, namely as $a^{-2/p}$. From the scaling solution (20) it is easy to see that the time-dependent part of the Kasner exponents β_i scales as $(-t)^{1-3p}$ so that these exponents approach constant values during the ekpyrotic phase where $p < \frac{1}{3}$. This gives rise to the so-called cosmic no-hair theorem for ekpyrosis [36], according to which a contracting universe with anisotropy and inhomogeneous curvature converges to a homogeneous, flat and isotropic universe if it contains energy with equation of state $w > 1$. From (14) and (22) we can see that the relative importance of the anisotropy term in the Friedmann equation drops by a factor of at least

$$\frac{(a^6 H^2)_{beg}}{(a^6 H^2)_{end}} \geq e^{120} \quad (30)$$

during the ekpyrotic phase (note that the relative importance of the curvature term $\propto a^{-2}H^{-2}$ drops off by a similar amount). Thus, at the end of the ekpyrotic phase, the size of pre-ekpyrotic anisotropies and spatial curvatures has become exponentially small!

The ekpyrotic phase does not last right up to the big crunch. Indeed, on general grounds one expects the potential to turn off and become irrelevant as the big crunch is neared, see section 6. Then the universe enters a kinetic energy dominated phase, with equation of state $w = 1$. The equations of motion reduce to

$$3H^2 = \frac{1}{2}\dot{\phi}^2 = -\dot{H} \quad \ddot{\phi} + 3H\dot{\phi} = 0. \quad (31)$$

They immediately lead to $a \propto e^{\phi/\sqrt{6}}$ and are solved by

$$a = a_0(-t)^{1/3} \quad \phi = \sqrt{\frac{2}{3}} \ln(-t) + \phi_0, \quad (32)$$

for some integration constants a_0, ϕ_0 . As will be derived in section 4, the ekpyrotic phase lasts until about 10^3 Planck times before the big crunch; the subsequent kinetic phase is relevant until the quantum gravity regime is reached around $t \sim -1M_{Pl}^{-1}$, and also again in the first moments after the big bang. Now of course one has to answer the question of whether this kinetic phase undoes what the ekpyrotic phase has achieved. From the solution above, one can see immediately that $\dot{\phi}^2 \propto a^{-6}$, which means that the scalar field energy density scales in the same way as the anisotropies. Thus the relative importance of the anisotropic term remains constant during the kinetic phase. In a similar way [36], one can show that the curvature terms are harmless during this $w = 1$ phase. In the presence of p-forms, depending on their coupling to the scalar ϕ , dangerous modes can grow, but only as a power of time t and for not longer than about 7 e-folds. In the relevant case of heterotic M-theory for example [73], one such coupling is critical [36] implying that the approach to the big crunch could be chaotic if there was sufficient energy density in the p-form modes.

Now it is clear how the ekpyrotic phase can avoid the uncertainties of a chaotic crunch: even when dangerous modes are present their energy density gets diluted to such an extent during ekpyrosis that they don't have time to come to dominate before less than a Planck time before the big crunch⁴ [36]. Thus chaos is avoided by being delayed until we have entered the quantum gravity regime, where the equations of general relativity break down. At that point, one might heuristically expect the spread of the wavefunction of the universe to prevent chaotic behavior from resurfacing. This last speculation will be discussed further in section 6.

3.2 *The Milne Universe*

After the ekpyrotic phase, the universe enters a kinetic energy dominated phase as it approaches the big crunch. Thus, from a 4-dimensional point of view, the universe seems to be headed towards the worst kind of singularity. However, in approaching the crunch at $t = 0$, the universe really starts looking 5-dimensional, and the higher-dimensional description reveals that the singularity is actually much milder. We will give a simplified description here; a more complete treatment is postponed to section 6.

Consider gravity in 5 dimensions, but assume that one spatial dimension is a line segment, as suggested by Hořava-Witten theory (see section 6). This line segment can be described as a circle modded out by a \mathbb{Z}_2 reflection symmetry across an axis, see Fig. 3. Then we can write the metric as [47]

⁴ What's more, some dangerous modes are further suppressed by the topology of the internal manifold, see [107].

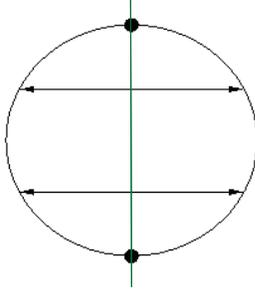


Fig. 3. A line segment represented as the orbifold S^1/\mathbb{Z}_2 . The thick dots represent the orbifold fixed points.

$$ds_5^2 = e^{-\sqrt{2/3}\phi} ds_4^2 + e^{2\sqrt{2/3}\phi} dy^2, \quad (33)$$

where $-y_0 < y < y_0$ denotes the orbifold coordinate and ϕ is the radion field parameterizing the size of the orbifold. The ϕ -dependent prefactor in front of the 4-dimensional metric ds_4^2 ensures that after dimensional reduction we are left with the canonically normalized Lagrangian

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 \right]. \quad (34)$$

We know that the kinetic phase described in equation (32) is a solution to this theory. This means that we can now plug the solution (32) into the original 5-dimensional metric, to find (up to trivial rescalings)

$$ds_5^2 = (-t)^{-2/3} [-dt^2 + (-t)^{2/3} dx_3^2] + (-t)^{4/3} dy^2 \quad (35)$$

$$= -dT^2 + T^2 dy^2 + dx_3^2, \quad (36)$$

after changing coordinates to $T \propto (-t)^{2/3}$. This spacetime goes by the name of “compactified Milne mod \mathbb{Z}_2 ” $\times \mathbb{R}_3$ [99]. It describes the two orbifold planes (*i.e.* the $(3+1)$ -dimensional boundaries of the line segment) approaching each other, colliding, and receding away from each other again, as shown in Fig. 4. Under the further change of coordinates $u = T \cosh y$, $v = T \sinh y$, the metric becomes

$$ds_5^2 = -du^2 + dv^2 + dx_3^2, \quad (37)$$

which shows that the embedding spacetime is simply Minkowski space. Thus, except at the collision itself, this spacetime is flat and therefore automatically a solution of any theory of gravity without a cosmological constant. This also means that higher-derivative corrections are small for a spacetime that is close to Milne. In this sense the compactified Milne space is the simplest model spacetime for a brane collision. Note that, from the 5-dimensional point of view, the big crunch singularity is just the momentary shrinking away of the orbifold dimension, while the other dimensions remain at finite size. This can best be seen by examining equation (33) together with the solution (32): even though the 4-dimensional scale factor $a \rightarrow 0$ at the collision, the brane

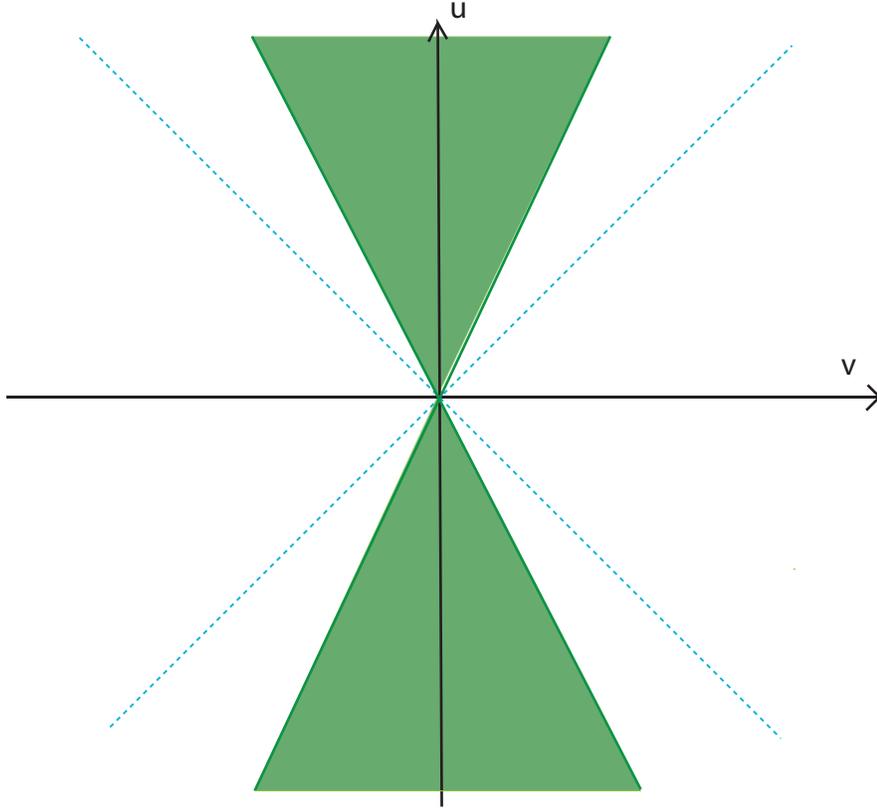


Fig. 4. The compactified Milne mod \mathbb{Z}_2 space describes the collision of two boundary branes, embedded in Minkowski space.

scale factors are really given by $e^{-\phi/\sqrt{6}}a$, which is a constant. Thus the density of matter (which is stuck on the branes) and the temperature remain finite at the collision.

The general idea here is that the ekpyrotic phase drives the universe to being exponentially flat and isotropic in a 4-dimensional sense, which means that the subsequent kinetic phase and big crunch/big bang transition can be well approximated in 5 dimensions by the compactified Milne spacetime. Of course one can only trust the spacetime description up to about a Planck time on either side of the collision. For a proper treatment, we would need a complete theory of quantum gravity. But in a sense, the ekpyrotic phase prepares the universe in the best possible state to enter the collision phase.

There have been a number of studies in string/M-theory of the Milne spacetime near the collision: Turok *et al.* [104] have shown how the equations for winding membranes remain well behaved near the collision. And in [82] it was argued that from the string theory perspective strings evolve as sets of weakly coupled “string bits” smoothly across the singularity. However it is not clear to what extent these treatments encompass all the relevant degrees of freedom near the collision. Furthermore, it was argued in [104] that near the collision the theory

should be described in terms of an expansion in the string tension $1/\alpha'$ rather than in α' ; such an expansion is not well understood at present. It is also known that perturbative string theory breaks down in these and similar settings as soon as interactions are taken into account [71,13,27]. But again it is not clear that perturbative string theory is really the right approach. The most promising development so far is the treatment of a toy model crunch/bang transition in the framework of the AdS/CFT correspondence by Craps *et al.* [103,26]. However, in this toy model, the bulk spacetime is 5-dimensional and of Kasner type close to the crunch. But most importantly, the radius of curvature of the AdS spacetime is smaller than the string length, in sharp contrast to ekpyrotic models, in which we rather expect the curvature to be small in approaching the crunch. Hence the results of this work cannot directly be applied to ekpyrotic models. Nevertheless, one might hope that the techniques used in [26] can be extended in the future to include the treatment of more realistic spacetimes.

Of course this is a topic of crucial importance for ekpyrotic theory. In order to discuss cosmological perturbation theory, it is vital to know how to match the universe going into the big crunch to the one coming out of the big bang. In light of the ambiguities discussed above, two broadly different attitudes can be adopted (see [10,20,29,34,45,74,76,77,98,99,102] for many discussions of these “matching conditions” in the literature):

In analogy with the treatment of reheating in inflation, Creminelli *et al.* [29] have advocated the model-independent assumption that the spacetime metric can simply go through the collision essentially unchanged. This assumption is based on the following argument: the ekpyrotic phase renders the universe isotropic up to exponentially small terms. Also, as we will see in the next section, quantum effects lead to small anisotropies in the metric. Thus, in synchronous gauge ($g_{0\mu} = \eta_{0\mu}$), the metric reads

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x^\mu)} e^{2h_{ij}(x^\mu)} dx^i dx^j, \quad (38)$$

where we are allowing for scalar curvature perturbations ζ and tensor perturbations h_{ij} . Then locally the small, long-wavelength perturbations we're interested in can be gauged away by a local re-scaling of the spatial coordinates. In other words, locally these fluctuations are just integration constants. Thus the universe undergoes the same history at every point in space up to exponentially small terms. If we then assume that the dynamics is not sensitive to these exponentially small terms (on super-horizon scales) then the metric will reemerge from the crunch/bang transition with the same integration constants, *i.e.* with the same perturbations. Note that this argument, if correct, applies not only to linear perturbations but to the full non-linear metric.

The second possibility is that the dynamics of the crunch/bang transition is

important: for example it was shown in [98,99] that for free fields in a compactified Milne universe, one can simply analytically continue the evolution of the fields around the singularity at $t = 0$. Such a prescription corresponds to matching perturbations on surfaces of constant energy density (see also [49,105]) and it leads to a mixing of different perturbation modes, such as the dominant and subdominant components of the Newtonian potential and the curvature perturbation. As the energy density perturbation is exponentially small, this represents a concrete example of a situation in which the history of the universe does depend significantly on the exponentially small perturbation terms. It should be noted however that this treatment breaks down once interactions are taken into account, as noted above. It has also been suggested by McFadden *et al.* [78] that a mixing of modes can occur when the effective 4-dimensional description breaks down near the collision. It should be clear that in this case as well, the evolution of the metric and its perturbations depend sensitively on the detailed dynamics of the bounce.

We will adopt the “nothing happens” assumption of Creminelli *et al.* in this review (*i.e.* we will assume that the curvature perturbation is non-linearly conserved through the collision), but with the proviso that this assumption is subject to revision once the crunch/bang transition is understood in more detail.

3.3 New Ekpyrotic Models

The idea behind the “new ekpyrotic” models [22,23,30] is to build cosmological models in which the ekpyrotic contracting phase is joined with the standard expanding phase by a smooth, non-singular bounce that can be described entirely within a 4-dimensional effective field theory. Thus, the energy never reaches the Planck scale, and cosmological perturbations can be evolved unambiguously through the bounce phase. Of course, in light of the singularity theorems of Hawking and Penrose, it is clear that in order to achieve a smooth bounce, one of the assumptions going into these theorems has to give; in the models considered so far this has been the null energy condition (NEC), which is violated by means of a ghost condensate [6].⁵ In the presence of a fluid with energy density ρ and pressure \mathcal{P} , the Friedmann equations can be combined to yield

$$\dot{H} = -\frac{1}{2}(\rho + \mathcal{P}). \quad (39)$$

The null energy condition requires that $\rho + \mathcal{P} \geq 0$, or, in covariant form, $T_{\mu\nu}n^\mu n^\nu \geq 0$ for any non-spacelike vector n^μ . Thus, in order to have $\dot{H} >$

⁵ We should note though that it is not clear yet if such a ghost condensate can be realized in string theory [3] or in a UV complete theory in general [46].

0, which is a necessary condition to revert from a contracting phase to an expanding one, the NEC must be violated.

We will briefly review the main ideas of ghost condensation, in order to see how the NEC can be violated: we start by considering an effective Lagrangian of the form

$$\mathcal{L} = \sqrt{-g}M^4P(X), \quad (40)$$

where

$$X = -\frac{1}{2m^4}(\partial\phi)^2 \quad (41)$$

and $P(X)$ is a function that remains to be specified. M and m are two mass scales that must be determined by the underlying microscopic theory, and, as we will see, that have to satisfy certain consistency conditions. Note that the theory admits the constant shift symmetry $\phi \mapsto \phi + \text{constant}$. In a cosmological background, the resulting equation of motion reads

$$\frac{d}{dt}(a^3P_{,X}\dot{\phi}) = 0, \quad (42)$$

which is automatically solved at an extremum $P_{,X} = 0$. We will assume that the extremum lies at $X_0 = \frac{1}{2}$ say, which in turn can be solved by giving the scalar ϕ the time-dependent expectation value

$$\phi = -m^2t. \quad (43)$$

With this expression for ϕ , we are justified *a posteriori* to have omitted higher-derivative terms like $\square\phi$ in the Lagrangian, since these would vanish in any case.

The ghost condensate by itself does not violate the null energy condition yet; in fact it has the same equation of state as a cosmological constant. To see this, note that the energy-momentum tensor is given by

$$T_{\mu\nu} = M^4P g_{\mu\nu} + \frac{M^4}{m^4}P_{,X}\partial_\mu\phi\partial_\nu\phi. \quad (44)$$

Thus the energy density is given by $\rho = M^4(2P_{,X}X - P)$, while the pressure is $\mathcal{P} = M^4P$. Hence, at the extremum, we have $\mathcal{P} = -\rho$ or $w = -1$. However, the fluctuations around the extremum can violate the NEC. We define the fluctuations π via

$$\phi = -m^2t + \pi. \quad (45)$$

Their dynamics can be expressed by the effective Lagrangian

$$\mathcal{L} \propto 2X_0P_{,XX}(X_0)\dot{\pi}^2, \quad (46)$$

which shows that the kinetic term has the correct sign if the extremum is a minimum, as we will assume (in other words, there are no ghosts around the

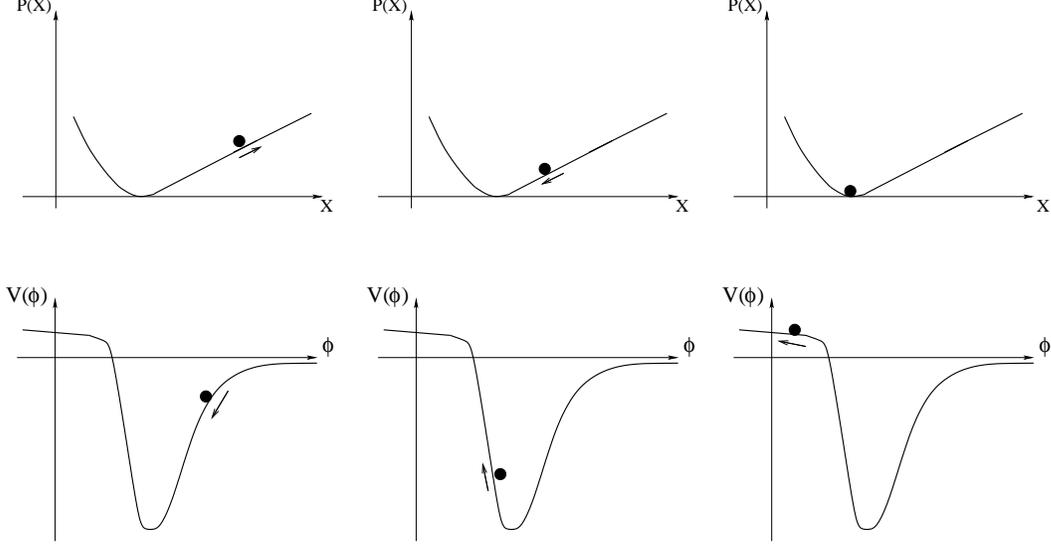


Fig. 5. This figure shows the evolution of the kinetic function $P(X)$ and of the potential $V(\phi)$ during the phases of ekpyrosis and bounce. Note that the field starts at approximately zero potential energy and moving very slowly, so that $\rho \sim -\dot{\pi} + V \approx 0$. Then, during the ekpyrotic phase, the potential is negative, implying that $\dot{\pi} < 0$ and hence $\dot{H} < 0$. Subsequently, during the ghost condensate phase, the potential shoots back up to positive values, implying that $\dot{\pi} > 0$ and thus $\dot{H} > 0$, *i.e.* the universe starts reverting from contraction to expansion. Figure reproduced with permission from [22].

ghost condensate). If we then expand the expression for the energy density, $\rho = M^4(2P_{,X}X - P)$, to linear order in π , we obtain $\rho \approx -KM^4\dot{\pi}/m^2$. We can provoke such a fluctuation by adding a potential V , implying that the energy density and the pressure are now approximately given by

$$\rho \approx -\frac{KM^2}{m^2}\dot{\pi} + V \quad \mathcal{P} \approx -V, \quad (47)$$

where $K \equiv P_{,XX}(X_0) > 0$ and $P(X_0) = 0$. From (39) we can see that this immediately implies

$$\dot{H} \approx \frac{KM^4}{2m^2}\dot{\pi}, \quad (48)$$

and hence, since $\dot{\pi}$ can take either sign, we now have the possibility of violating the NEC.

Nevertheless, it is still difficult to achieve a bounce because NEC-violating modes scale with a positive power of the scale factor, see equation (6). Hence they tend to become subdominant in a contracting universe, and dominant in an expanding one - exactly the opposite of what we want here! This problem may be circumvented if it is the same field driving both the ekpyrotic phase and the bounce. For this to work though, we need the Lagrangian function $P(X)$ to be canonical (linear) during ekpyrosis [22] (although see [30] for a different approach), and then quadratic around the minimum; also, the potential V

has to become positive at the onset of the bounce phase in order to push the equation of state parameter w to NEC-violating values below -1 . The complete evolution is sketched in Fig. 5. After the bounce is complete, it is assumed that the NEC-violating fluid decays by converting all its energy into matter and radiation, and that it thereby reheats the universe [22].

In these models, there are two sources of instability, which must be addressed. One is of the Jeans type and there is also a gradient instability, see [22,28,30] for details. Both of these instabilities are harmless if the bounce phase is rather fast, *i.e.* if it lasts only on the order of one Hubble time (as measured at the start of the bounce phase). In that case, there isn't enough time for the instabilities to grow. A further constraint that these models have to satisfy in order to obtain a successful bounce is that the value of the potential at the end of the ekpyrotic phase has to fall within a certain range, so that the ekpyrotic phase lasts long enough and so that the trajectory ends up near the ghost-condensate point at the end of ekpyrosis. As discussed in [23], this bound is

$$m^4 e^{2\mathcal{N}_{ek}} \ll |V_{end}| \ll \frac{M^4 K}{p}, \quad (49)$$

where $\mathcal{N}_{ek} = \frac{1}{2} \ln |V_{end}/V_{beg}|$ is the number of e-folds of the ekpyrotic phase; in other words, we must require that M is exponentially larger than m . Of course it would be satisfying to see how this large hierarchy in the effective theory could arise from an underlying, more fundamental, theory.

But this is not our direct concern here. The new ekpyrotic models rather aim to represent a proof-in-principle that it is possible to have a cosmological history that can be described entirely within the framework of a 4-dimensional effective theory, so that the evolution of all fields, and in particular of cosmological perturbations, can be followed unambiguously through the bounce. We will return to the issue of cosmological perturbations during the bounce phase in the next section.

4 Cosmological Perturbations

The ekpyrotic phase not only manages to address the standard cosmological puzzles, but it also provides a new way of generating cosmological perturbations. Both scalar and tensor perturbations are generated, but, as we will see, they turn out to have very different properties. The colliding branes picture provides a heuristic way of understanding the origin of the perturbations [48]: although the ekpyrotic phase renders the branes very flat and parallel classically, quantum fluctuations cause the branes to ripple slightly. Thus, the branes do not collide everywhere at exactly the same time, and therefore, in some places the big bang happens slightly earlier than in others. In this way,

some places have a little bit more time to expand and cool than others, and the result is a pattern of temperature fluctuations in the cosmic microwave background (CMB). We will now make this cartoon quantitatively precise [48,49,39,18].

4.1 Scalar perturbations

The detailed shape of the angular anisotropy power spectrum of the CMB radiation can be described by fluctuation modes that enter the horizon in succession, *i.e.* in a timed, coordinated manner. This hints to having a period of cosmological evolution where the modes were in causal contact initially, and then went outside of the horizon, only to re-enter later in succession. There are, broadly speaking, two ways of producing this phenomenon: in inflation, the horizon is roughly constant and the modes get stretched rapidly beyond it, whereas in ekpyrosis, due to the increasing ultralocality in approaching the crunch, the horizon shrinks rapidly while the fluctuation modes themselves remain roughly constant in size. As we will see, for scalar, linear perturbations this leads to a kind of duality between the two theories. Tensor modes, on the other hand, are sensitive only to the evolution of the metric, and hence behave very differently in the two theories; this will be discussed in section 4.2.

4.1.1 Single Field

We will start with a simple, heuristic argument for why one might expect to obtain scale-invariant perturbations in the presence of a scalar field with a steep negative potential [61]. In fact, since the universe is contracting very slowly during the ekpyrotic phase, to a first approximation we can ignore gravity altogether. The validity of this approximation will be discussed shortly. So, for now consider a scalar field ϕ in Minkowski spacetime, with action

$$\mathcal{S} = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + V_0 e^{-c\phi} \right). \quad (50)$$

We consider the case where V_0 is positive so the potential energy is formally unbounded below and the scalar field runs to $-\infty$ in a finite time. Note that the actual value of V_0 is not a physical parameter, since by shifting the field ϕ one can alter the value of V_0 arbitrarily. Furthermore, in the ekpyrotic or cyclic models, the potential is expected to turn up towards zero at large negative ϕ (because this corresponds to the limit in heterotic M-theory in which the string coupling approaches zero and the potential disappears [93]), but this detail is irrelevant to the generation of perturbations on long wavelengths.

We now argue that, as the background scalar field rolls down the exponential

potential towards $-\infty$, then, to leading order in \hbar , its quantum fluctuations acquire a scale-invariant spectrum as the result of three features. First, the action (50) is classically scale-invariant. Second, by re-scaling $\phi \rightarrow \phi/c$ and re-defining V_0 , the constant c can be brought out in front of the action and absorbed into Planck's constant \hbar , *i.e.* $\hbar \rightarrow \hbar/c^2$, in the expression $i\mathcal{S}/\hbar$ governing the quantum theory. Finally, it shall be important that ϕ has dimensions of mass in four spacetime dimensions.

To see the classical scale-invariance, note that shifting the field $\phi \rightarrow \phi + \psi$, and re-scaling coordinates $x^\mu \rightarrow x^\mu e^{c\psi/2}$, just re-scales the action by $e^{c\psi}$ and hence is a symmetry of the space of solutions of the classical field equations. Now we consider a spatially homogeneous background solution corresponding to zero energy density in the scalar field. In a cyclic model (see section 5), this “zero energy” condition is a reasonable initial state to assume for analyzing perturbations because the phase in which the perturbations are generated is preceded by a very low energy density phase with an extended period of accelerated expansion, like that of today's universe, which drives the universe into a very low energy, homogeneous state. No new energy scale enters and the solution for the scalar field is then determined (up to a constant) by the scaling symmetry: $\phi_b = (2/c) \ln(-At)$. Next we consider quantum fluctuations $\delta\phi$ in this background. The classical equations are time-translation invariant, so a spatially homogeneous time-delay is an allowed perturbation, $\phi = (2/c) \ln(-A(t + \delta t)) \Rightarrow \delta\phi \propto t^{-1}$. On long wavelengths, for modes whose evolution is effectively frozen by causality, *i.e.* $|kt| \ll 1$, we can expect the perturbations to follow this behavior. Hence, the quantum variance in the scalar field is

$$\langle \delta\phi^2 \rangle \propto \hbar t^{-2}. \quad (51)$$

Restoring c via $\phi \rightarrow c\phi$ and $\hbar \rightarrow c^2\hbar$ leaves the result unchanged. However, since $\delta\phi$ has the same dimensions as t^{-1} in four spacetime dimensions, it follows that the constant of proportionality in (51) is dimensionless, and therefore that $\delta\phi$ must have a scale-invariant spectrum of spatial fluctuations.

It is straightforward to check this in detail. Setting $\phi = \phi_b(t) + \delta\phi(t, \mathbf{x})$, to linear order in $\delta\phi$ the field equation reads

$$\ddot{\delta\phi} = -V_{,\phi\phi} \delta\phi + \nabla^2 \delta\phi \quad (52)$$

Using the zero energy condition for the classical background, we obtain $V_{,\phi\phi} = c^2 V = -c^2 \dot{\phi}_b^2 / 2 = -2/t^2$. Next we set $\delta\phi(t, \mathbf{x}) = \sum_{\mathbf{k}} (a_{\mathbf{k}} \chi_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.)$ with $a_{\mathbf{k}}$ the annihilation operator and $\chi_{\mathbf{k}}(t)$ the normalized positive frequency modes. The mode functions χ_k obey

$$\ddot{\delta\chi}_k = \frac{2}{t^2} \chi_k - k^2 \chi_k, \quad (53)$$

and the incoming Minkowski vacuum state corresponds to [14]

$$\chi_k = \frac{1}{\sqrt{2k}} e^{-ikt} \left(1 - \frac{i}{kt} \right). \quad (54)$$

For large $|kt|$, this solution tends to the usual Minkowski positive-frequency mode. But as $|kt|$ tends to zero, each mode enters the growing time-delay solution described above, with $\chi_k \propto t^{-1}$. Computing the variance of the quantum fluctuation and subtracting the usual Minkowski spacetime divergence, we obtain

$$\langle \delta\phi^2 \rangle = \hbar \int \frac{k^2 dk}{4\pi^2} \frac{1}{k^3 t^2}, \quad (55)$$

where the integral is taken over k modes which have “frozen in” to follow the time-delay mode. In agreement with the general argument above, we have obtained a scale-invariant spectrum of growing scalar field perturbations. To recap, classical scale-invariance determines the t dependence of the perturbations, and dimensional analysis then gives a scale-invariant spectrum in k , in three space dimensions.

However, it is important to realize that what is measured in the CMB is directly related to the curvature perturbation of the spacetime geometry. Hence it is imperative that we include gravity in the analysis, even though *a priori* its effects appear to be small corrections. As we will see, the inclusion of gravity entails some (perhaps unexpected) subtleties [29]. Thus, we now consider the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 + V_0 e^{-c\phi} \right]. \quad (56)$$

The resulting equations of motion (the one-field versions of (17) and (18)) are solved by the one-field scaling solution (20). It is convenient to define the parameter ϵ via

$$\epsilon \equiv \frac{3}{2}(1+w). \quad (57)$$

In inflation this is the slow-roll parameter; here, since the potential is steep, $w \gg 1$ and ϵ rather represents “fast-roll”. In the scaling solution, it is simply given by $\epsilon = 1/p$. It is also useful to continue the following analysis in terms of conformal time τ , defined via $dt = a d\tau$. We will denote derivatives w.r.t conformal time by a prime, *i.e.* $\frac{d}{d\tau} \equiv '$. In terms of conformal time, the scaling solution (20) reads (up to unimportant re-scalings)

$$a = (-\tau)^{p/(1-p)}, \quad \phi_i = \frac{2}{c_i(1-p)} \ln(-\tau), \quad p = \sum_i \frac{2}{c_i^2}. \quad (58)$$

The single-field scalar perturbation theory was first discussed in [48,49]. In the next few paragraphs, we will closely follow the presentation of [18]. Including scalar perturbations, the metric can be written as

$$\begin{aligned}
ds^2/a^2 = & -(1 + 2AY)d\tau^2 - 2BY_i d\tau dx^i \\
& + [(1 + 2H_L Y)\delta_{ij} + 2H_T Y_{ij}] dx^i dx^j
\end{aligned} \tag{59}$$

and the perturbed scalar field as

$$\phi = \phi_0(\tau) + \delta\phi(\tau)Y \tag{60}$$

where $Y(\vec{x})$, $Y_i(\vec{x})$, and $Y_{ij}(\vec{x})$ are scalar harmonics. Here we work in Fourier space, so that we are not writing out the implicit subscripts \vec{k} of perturbation modes. The perturbed Einstein equations $\delta G^\mu{}_\nu = \delta T^\mu{}_\nu$ then relate the metric and the matter perturbations to each other. As is well known, scalar perturbations in a spatially-flat FRW universe with scalar field ϕ and potential $V(\phi)$ are completely characterized by a single gauge-invariant variable. Various choices for this variable are typically considered, including the ‘‘Newtonian potential’’ Φ , and the ‘‘curvature perturbation’’ ζ . The gauge-invariant Newtonian potential Φ is most easily understood in ‘‘Newtonian gauge’’ ($B = H_T = 0$), where it is related to the metric perturbations in a simple way: $\Phi = A = -H_L$. It obeys the equation of motion

$$\Phi'' + 2 \left[\frac{a'}{a} - \frac{\phi_0''}{\phi_0'} \right] \Phi' + 2 \left[k^2 + 2\mathcal{H}' - 2\mathcal{H} \frac{\phi_0''}{\phi_0'} \right] \Phi = 0 \tag{61}$$

where $k = |\vec{k}|$ is the magnitude of the (comoving) Fourier 3-vector. On the other hand, the gauge-invariant perturbation variable ζ is most easily understood in ‘‘comoving gauge’’ ($H_T = \delta T^0{}_i = 0$), where it represents the curvature perturbation on spatial hypersurfaces, and is related to the spatial metric perturbation in a simple way: $\zeta = -H_L$. The condition $\delta T^0{}_i = 0$ also implies that $\delta\phi = 0$ in this gauge. ζ obeys the equation of motion

$$\zeta'' + 2\frac{z'}{z}\zeta' + k^2\zeta = 0, \tag{62}$$

where we have defined $z \equiv a^2\phi_0'/a'$. Φ and ζ are related to each other by

$$\zeta = \Phi + \frac{1}{\epsilon} \left[\frac{a}{a'} \Phi' + \Phi \right] \tag{63}$$

$$\Phi = -\epsilon \frac{a'}{ak^2} \zeta'. \tag{64}$$

It is convenient to introduce new variables, u and v [81] (these are of course unrelated to the coordinates used in section 3.2), by multiplying Φ and ζ by k -independent functions of τ :

$$u \equiv \frac{a}{\phi_0'} \Phi \quad v \equiv z\zeta. \tag{65}$$

Note that u and v have the same k -dependence as Φ and ζ , respectively, and therefore the same spectral properties. u and v obey the simple equations of motion

$$u'' + \left(k^2 - \frac{(1/z)''}{(1/z)}\right)u = 0 \quad (66)$$

$$v'' + \left(k^2 - \frac{z''}{z}\right)v = 0, \quad (67)$$

and are related to each other by

$$kv = 2k\left(u' + \frac{z'}{z}u\right) \quad (68)$$

$$-ku = \frac{1}{2k}\left(v' + \frac{(1/z)'}{1/z}v\right). \quad (69)$$

When ϵ is time-independent, we can use (20) to find

$$\frac{(1/z)''}{1/z} = \frac{\epsilon}{(\epsilon - 1)^2\tau^2} \quad (70)$$

$$\frac{z''}{z} = \frac{2 - \epsilon}{(\epsilon - 1)^2\tau^2} \quad (71)$$

Thus, at early times (when $|\tau|$ is large) the k^2 term dominates the $(1/z)''/(1/z)$ and z''/z terms in the brackets in the equations of motion (66) and (67). Physically, this means that the relevant modes are well within the horizon, where the solutions are asymptotically oscillatory. On scales much smaller than the horizon, the curvature of spacetime is negligible, and thus we impose boundary conditions corresponding to the Minkowski vacuum seen by a coming observer [14]:

$$u \rightarrow \frac{i}{(2k)^{3/2}}e^{-ik\tau} \quad (72)$$

$$v \rightarrow \frac{1}{\sqrt{2k}}e^{-ik\tau} \quad (73)$$

as $\tau \rightarrow -\infty$.

The equations of motion are Bessel equations, and they can be solved exactly by

$$u(x) = x^{1/2} \left[A^{(1)} H_\alpha^{(1)}(x) + A^{(2)} H_\alpha^{(2)}(x) \right] \quad (74)$$

$$v(x) = x^{1/2} \left[B^{(1)} H_\beta^{(1)}(x) + B^{(2)} H_\beta^{(2)}(x) \right] \quad (75)$$

where $x \equiv k|\tau|$ is a dimensionless time variable, $A^{(1,2)}$ and $B^{(1,2)}$ are constants, $H_s^{(1,2)}(x)$ are Hankel functions, and we have defined

$$\alpha \equiv \sqrt{\frac{(1/z)''}{1/z}\tau^2 + 1/4} = \frac{1}{2} \left| \frac{\epsilon + 1}{\epsilon - 1} \right| \quad (76)$$

$$\beta \equiv \sqrt{\frac{z''}{z}\tau^2 + 1/4} = \frac{1}{2} \left| \frac{\epsilon - 3}{\epsilon - 1} \right| \quad (77)$$

In the far past ($x \rightarrow \infty$) we use the asymptotic Hankel expression

$$H_s^{(1,2)}(x) \rightarrow \sqrt{\frac{2}{\pi x}} \exp \left[\pm i \left(x - \frac{s\pi}{2} - \frac{\pi}{4} \right) \right] \quad (78)$$

so that the boundary conditions pick out the solutions

$$u = \frac{\mathcal{P}_1}{2k} \sqrt{\frac{\pi x}{4k}} H_\alpha^{(1)}(x) \quad (79)$$

$$v = \mathcal{P}_2 \sqrt{\frac{\pi x}{4k}} H_\beta^{(1)}(x) \quad (80)$$

with the phase factors

$$\mathcal{P}_1 = \exp[i(2\alpha + 3)\pi/4] \quad (81)$$

$$\mathcal{P}_2 = \exp[i(2\beta + 1)\pi/4]. \quad (82)$$

In order to determine the power spectra of ζ and Φ , at late times when co-moving scales are well outside Hubble radius, we need the asymptotic form of the Hankel functions as $x \rightarrow 0$:

$$H_s^{(1)}(x) \rightarrow -\frac{i}{\pi} \Gamma(s) \left(\frac{x}{2}\right)^{-s} \quad (83)$$

where $s > 0$ and $\Gamma(s)$ is the Euler gamma function. On large scales, the power spectrum of the Newtonian potential Φ is given by

$$P_\Phi(k) = \frac{k^3}{2\pi^2} \frac{|u|^2 \phi_0^2}{a^2} \propto x^{1-2\alpha}. \quad (84)$$

The spectral index for Φ is thus

$$n_\Phi - 1 = 1 - 2\alpha = 1 - \left| \frac{\epsilon + 1}{\epsilon - 1} \right|. \quad (85)$$

Note that it is invariant under $\epsilon \rightarrow 1/\epsilon$, which is a manifestation (at the linear level) of a duality between inflation ($\epsilon \approx 0$) and ekpyrosis ($\epsilon \gg 1$) [18]. In fact, this could have been anticipated on the basis of the invariance

of both the equation of motion and the boundary condition for u under the transformation $\epsilon \rightarrow 1/\epsilon$. Note that in both limits of interest, namely $\epsilon \rightarrow 0$ (inflation) and $\epsilon \rightarrow \infty$ (ekpyrosis), the spectrum of the Newtonian potential turns out to be scale-invariant. If we had performed the same calculation, but with an equation of state that is slowly varying in time, then we would have obtained the spectral index [39]

$$n_\Phi - 1 = -4(\bar{\epsilon} + \bar{\eta}), \quad (86)$$

where

$$\bar{\epsilon} = \left(\frac{V}{V_{,\phi}}\right)^2, \quad \bar{\eta} = 1 - \frac{VV_{,\phi\phi}}{V_{,\phi}^2}. \quad (87)$$

On large scales, the power spectrum for ζ is given by

$$P_\zeta(k) = \frac{k^3}{2\pi^2} \frac{|v|^2}{z^2} \propto x^{3-2\beta}, \quad (88)$$

which means that the spectral index for ζ is

$$n_\zeta - 1 = 3 - 2\beta = 3 - \left|\frac{\epsilon - 3}{\epsilon - 1}\right|. \quad (89)$$

In ekpyrotic models, ϵ is large, and thus the spectrum of the curvature perturbation ζ is very blue, meaning that there is much more power on smaller scales, in disagreement with observations.

This seems very puzzling at first, since one would have thought that it didn't matter whether one used Φ or ζ in order to perform the calculation. However, it is instructive to rewrite the relationship (63) between the two variables in the following form:

$$\zeta = \frac{1}{\epsilon a^2} \left(\frac{a^3 \Phi}{a'}\right)'. \quad (90)$$

Expanding the Hankel function $H^{(1)}(x)$ again as $x \rightarrow 0$, but this time up to second order, one finds

$$H_s^{(1)}(x) = -\frac{i}{\pi} \Gamma(s) \left(\frac{x}{2}\right)^{-s} - \frac{i}{\pi} e^{-i\pi s} \Gamma(-s) \left(\frac{x}{2}\right)^s + O(x^{2-s}, x^{2+s}), \quad (91)$$

which means that Φ behaves roughly speaking as

$$\Phi \sim k^{-3/2-1/\epsilon} (-\tau)^{-1-2/\epsilon} + k^{-1/2+1/\epsilon}, \quad (92)$$

where we have expanded all exponents up to linear order in $1/\epsilon$. But using the same expansion, we have that

$$\frac{a'}{a^3} \sim (-\tau)^{-1-2/\epsilon}, \quad (93)$$

so that from (90) we can see that the leading, scale-invariant contribution to Φ drops out of ζ !

Here we see very clearly the importance of adding gravity to our analysis. In light of the discussion in section (3.2) this means that the single-field ekpyrotic model does not lead to a scale-invariant spectrum of cosmological perturbations unless there is a mixing of Φ and ζ at the bounce (in that case, the scale-invariant component of Φ would dominate on large scales over the blue intrinsic spectrum of ζ). Such a mixing would depend on the precise dynamics at the bounce, and would therefore be a model-dependent prediction. Examples of mixing due to higher-dimensional effects have been given in [78,99], and mixing can occur for example due to the breakdown of the 4-dimensional effective field theory. But what this means physically is that new degrees of freedom become relevant near the brane collision, and therefore it is rather natural to try to employ a more complete effective theory, containing more fields, in order to perform the above calculation. This will be the topic of the following section.

4.1.2 Two Fields

In a sense it is rather unnatural to consider only a single scalar field in the effective theory, since there are at least two “universal” fields that are always present in a higher-dimensional context: the radion field, determining the distance between the two end-of-the-world branes, and the volume modulus of the internal manifold. But as soon as there are more than one scalar field present, one can have entropy, or isocurvature, perturbations, which are growing mode perturbations in a collapsing universe [83]. Entropy perturbations can source the curvature perturbation, and hence (provided the entropy perturbations acquire a nearly scale-invariant spectrum), almost scale-free curvature perturbations can be generated just before the bounce. These then turn into growing mode perturbations in the ensuing expanding phase.

As will be shown in section 6, the effective theory arising from heterotic M-theory, which describes gravity and the two universal scalars mentioned above, is a 4-dimensional theory of gravity minimally coupled to two scalar fields (after a field redefinition). It is thus surprisingly simple. On top of that, we assume an attractive force between the boundary branes, modeled by two negative nearly exponential potentials for the scalar fields:

$$V = -V_1 e^{-\int c_1 d\phi_1} - V_2 e^{-\int c_2 d\phi_2}, \quad (94)$$

where $c_1 = c_1(\phi_1)$, $c_2 = c_2(\phi_2)$, and V_1 and V_2 are positive constants. We consider potentials in which the c_i are slowly varying and hence the potentials are locally exponential in form. Furthermore, for simplicity, we focus on scaling background solutions in which both fields *simultaneously* diverge to $-\infty$. The

background solution we are interested in is the two-field version of the scaling solution (20). We will find it useful to define the variable σ via [38]

$$\dot{\sigma} \equiv \sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}. \quad (95)$$

σ has the interpretation of being the path length along the background scalar field trajectory. The angle θ of the background trajectory is defined by [38] $\cos(\theta) = \dot{\phi}_1/\dot{\sigma}$, $\sin(\theta) = \dot{\phi}_2/\dot{\sigma}$. Then the fast-roll parameter ϵ and the equation of state parameter w are related to the background evolution via

$$\epsilon \equiv \frac{3}{2}(1+w) = \frac{\dot{\sigma}^2}{2H^2}. \quad (96)$$

During the phase in which the entropic perturbations are generated, $\dot{\theta} = 0$, which corresponds to a straight background trajectory in scalar field space. In this case, we define

$$\dot{\phi}_2 \equiv \gamma \dot{\phi}_1, \quad (97)$$

and, with this notation, $c_1 = \gamma c_2$ and

$$\epsilon_{ek} = \frac{|\gamma c_1 c_2|}{2(1+\gamma^2)}. \quad (98)$$

Thus we have an ekpyrotic phase as long as $c_1 c_2$ is large. γ is typically of $\mathcal{O}(1)$; for the heterotic M-theory colliding branes solution with empty branes, $\gamma = -\frac{1}{\sqrt{3}}$ during the ekpyrotic phase, as will be derived in section 6. It turns out though that the results are not very sensitive to γ .

The angle of the background trajectory is related to the potential via

$$\dot{\theta} = \frac{\dot{\phi}_2 V_{,\phi_1} - \dot{\phi}_1 V_{,\phi_2}}{\dot{\phi}_1^2 + \dot{\phi}_2^2}. \quad (99)$$

Hence, since we want a straight line trajectory, we have

$$V = \tilde{V}(\phi_1) + \gamma^2 \tilde{V}(\phi_2/\gamma), \quad (100)$$

for some function $\tilde{V}(\phi)$.

In a contracting universe, a growing mode is given by the entropy perturbation δs , namely the relative fluctuation in the two fields, defined (at linear order) as follows [38]

$$\delta s \equiv (\dot{\phi}_1 \delta \phi_2 - \dot{\phi}_2 \delta \phi_1)/\dot{\sigma}. \quad (101)$$

The entropy perturbation is gauge-invariant and it represents the perturbation orthogonal to the background scalar field trajectory, as shown in Fig. 6; see Ref. [58] for its definition to all orders.

The entropy perturbation equation of motion reads (see *e.g.* [38])

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2 \right) \delta s = \frac{4k^2\dot{\theta}}{a^2\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}} \Phi. \quad (102)$$

Successive derivatives of the potential with respect to the entropy field are given by

$$V_s = \frac{1}{\dot{\sigma}} (\dot{\phi}_1 V_{,\phi_2} - \dot{\phi}_2 V_{,\phi_1}) \quad (103)$$

$$V_{ss} = \frac{1}{\dot{\sigma}^2} (\dot{\phi}_1^2 V_{,\phi_2\phi_2} - 2\dot{\phi}_1\dot{\phi}_2 V_{,\phi_1\phi_2} + \dot{\phi}_2^2 V_{,\phi_1\phi_1}) \quad (104)$$

$$V_{sss} = \frac{1}{\dot{\sigma}^3} (\dot{\phi}_1^3 V_{,\phi_2\phi_2\phi_2} - 3\dot{\phi}_1^2\dot{\phi}_2 V_{,\phi_1\phi_2\phi_2} + 3\dot{\phi}_1\dot{\phi}_2^2 V_{,\phi_1\phi_1\phi_2} - \dot{\phi}_2^3 V_{,\phi_1\phi_1\phi_1}). \quad (105)$$

Incidentally, note that this implies $\dot{\theta} = -V_s/\dot{\sigma}$.

Again, for simplicity we will focus attention on straight line trajectories in scalar field space. Since $\dot{\theta} = 0$, the entropy perturbation is not sourced by the Newtonian potential Φ and we can solve the equations rather simply. On large scales, the linear entropy equation then reduces to

$$\ddot{\delta s} + 3H\dot{\delta s} + V_{ss}\delta s = 0. \quad (106)$$

It is convenient at this point to continue the analysis in terms of conformal time τ . Introducing the re-scaled entropy field

$$\delta S = a(\tau) \delta s, \quad (107)$$

Eq. (106) becomes

$$\delta S'' + \left(-\frac{a''}{a} + a^2 V_{,\phi_1\phi_1} \right) \delta S = 0. \quad (108)$$

The crucial term governing the spectrum of the perturbations is then

$$\tau^2 \left(\frac{a''}{a} - V_{,\phi_1\phi_1} a^2 \right). \quad (109)$$

When this quantity is approximately 2, we will get nearly scale-invariant perturbations. We proceed [61] by evaluating the quantity in (109) in an expansion in inverse powers of ϵ and its derivatives with respect to N , where $N = \ln(a/a_{end})$, where a_{end} is the value of a at the end of the ekpyrotic phase. Note that N decreases as the fields roll downhill and the contracting ekpyrotic phase proceeds.

We obtain the first term in (109) by differentiating (19), obtaining

$$\frac{a''}{a} = 2H^2 a^2 \left(1 - \frac{1}{2} \epsilon\right). \quad (110)$$

The second term in (109) is found by differentiating (96) twice with respect to time and using the background equations and the definition of N . We obtain

$$a^2 V_{,\phi_1\phi_1} = -a^2 H^2 \left(2\epsilon^2 - 6\epsilon - \frac{5}{2} \epsilon_{,N}\right) + O(\epsilon^0). \quad (111)$$

Finally, need to express $\mathcal{H} \equiv (a'/a) = aH$ in terms of the conformal time τ . From (110) we obtain

$$\mathcal{H}' = \mathcal{H}^2(1 - \epsilon), \quad (112)$$

which integrates to

$$\mathcal{H}^{-1} = \int_0^\tau d\tau(\epsilon - 1). \quad (113)$$

Now, inserting $1 = d(\tau)/d\tau$ under the integral and using integration by parts we can re-write this as

$$\mathcal{H}^{-1} = \epsilon\tau \left(1 - \frac{1}{\epsilon} - (\epsilon\tau)^{-1} \int_0^\tau \epsilon' \tau d\tau\right). \quad (114)$$

Using the same procedure once more, the integral in this expression can be written as

$$(\epsilon\tau)^{-1} \int_0^\tau \epsilon' \tau d\tau = \frac{\epsilon' \tau}{\epsilon} - (\epsilon\tau)^{-1} \int_0^\tau \frac{d}{d\tau}(\epsilon' \tau) \tau d\tau. \quad (115)$$

Now using the fact that $\epsilon' = \mathcal{H}\epsilon_{,N}$, and that to leading order in $1/\epsilon$, \mathcal{H} can be replaced by its value in the scaling solution (with constant ϵ), $\mathcal{H}\tau = \epsilon^{-1}$, we can re-write the second term on the right-hand side as

$$-(\epsilon\tau)^{-1} \int_0^\tau \frac{d}{d\tau}(\epsilon' \tau) \tau d\tau = -(\epsilon\tau)^{-1} \int_0^\tau d\tau \frac{1}{\epsilon} \left(\frac{\epsilon_{,N}}{\epsilon}\right)_{,N}, \quad (116)$$

which shows that this term is of order $1/\epsilon^2$ and can thus be neglected. Altogether we obtain

$$\mathcal{H}^{-1} = \int_0^\tau d\tau(\epsilon - 1) \approx \epsilon\tau \left(1 - \frac{1}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2}\right). \quad (117)$$

Using (110) and (111) with (117) we can calculate the crucial term entering the entropy perturbation equation,

$$\tau^2 \left(\frac{a''}{a} - V_{,\phi_1\phi_1} a^2\right) = 2 \left(1 - \frac{3}{2\epsilon} + \frac{3}{4} \frac{\epsilon_{,N}}{\epsilon^2}\right). \quad (118)$$

Following the same steps as in the single field section above, it is straightforward to see that the deviation from scale-invariance in the spectral index of

the entropy perturbation is now given by

$$n_s - 1 = \frac{2}{\epsilon} - \frac{\epsilon_{,N}}{\epsilon^2}. \quad (119)$$

The first term on the right-hand side is a gravitational contribution, which, being positive, tends to make the spectrum blue. The second term is a non-gravitational contribution, which tends to make the spectrum red. We will return to this expression shortly. Before explaining how these entropic perturbations are naturally converted to curvature perturbations, we must add an important comment: the growth of the entropy perturbations is intimately linked to an instability of the background trajectory [61,100]. Indeed, the background trajectory follows a ridge in the potential [57], and in order to achieve a sufficiently long ekpyrotic phase, the trajectory has to remain near the crest of the ridge for long enough. This places important constraints on the two-field models, which we will discuss in section 5.

We have shown how an approximately scale-invariant spectrum of entropy perturbations may be generated by scalar fields in a contracting universe. We will now discuss how these perturbations may be converted to curvature perturbations if the scalar field undergoes a sudden acceleration, and we will estimate the curvature perturbation amplitude. At linear order, a non-zero entropy perturbation combined with a bending ($\dot{\theta} \neq 0$) of the background trajectory sources the curvature perturbation on large scales and results in a linear, gaussian curvature perturbation (see *e.g.* [38])

$$\zeta_L = \int_{\Delta t} -\frac{2H}{\dot{\sigma}} \dot{\theta} \delta s^{(1)} \quad (120)$$

$$= \int_{\Delta t} \sqrt{\frac{2}{\epsilon_c}} \dot{\theta} \delta s^{(1)}, \quad (121)$$

where we denote the duration of the conversion by Δt (*i.e.* Δt is the time during which $\dot{\theta} \neq 0$) and the sign corresponds to a contracting universe. Thus the strength of conversion is proportional to $1/\sqrt{\epsilon_c}$ and this dependence on the equation of state during the time of conversion will have repercussions for the magnitude of the second order correction as well, as will be discussed in the next section. There are many ways in which such a bending of the background trajectory can occur. We will now discuss the various possibilities considered in the literature, namely conversion after the ekpyrotic phase during kinetic energy domination [61], conversion during the ekpyrotic phase [57,56] or during the transition to a ghost condensate phase [22,30], and conversion after the big bang by modulated preheating [9].

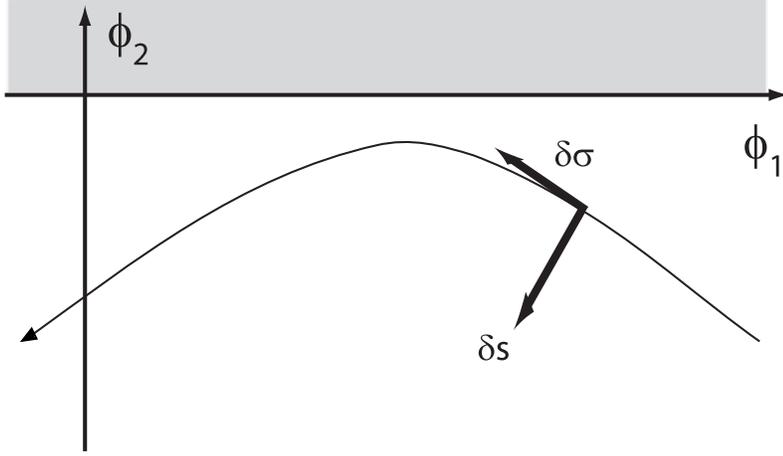


Fig. 6. The trajectory in field space reflects off a boundary at $\phi_2 = 0$. The entropy perturbation, denoted δs , is orthogonal to the trajectory. The bending causes the conversion of entropy modes into adiabatic modes $\delta\sigma$, which are perturbations tangential to the trajectory.

Conversion during kinetic energy domination

In the original ekpyrotic and cyclic models, the phase dominated by the steep, ekpyrotic potential $V(\phi)$ comes to an end (at $t = t_{end} < 0$) before the big crunch/big bang transition ($t = 0$), and the universe becomes dominated by the kinetic energy of the scalar fields, as we saw in section 3. Consequently, the equation of state at $t = t_{end}$ changes from $\epsilon_{ek} \gg 1$ to $\epsilon = 3$ (corresponding to $w \rightarrow 1$, the equation of state for a kinetic energy dominated universe). The conversion from entropic to curvature perturbations then takes place during this kinetic energy dominated phase [61]. The conversion occurs naturally in the heterotic M-theory embedding of the cyclic model because the negative-tension brane bounces off a spacetime singularity [67] – creating a bend in the trajectory in field space in the 4-dimensional effective theory – before it collides with the positive-tension brane (the big crunch/big bang transition). This will be explained in detail in section 6. Here, all we need to know is that in the 4-dimensional effective description there are two scalar field moduli, ϕ_1 and ϕ_2 , living on the half-plane $-\infty < \phi_1 < \infty$, $-\infty < \phi_2 < 0$. In other words, there is a boundary to moduli space at $\phi_2 = 0$. Furthermore, the cosmological solution of interest is one in which ϕ_2 encounters this boundary $\phi_2 = 0$, and reflects off it at time $t = t_{ref}$; see Figure 6. This bending of the trajectory automatically induces the conversion of entropy to curvature perturbations. Here we will not restrict the analysis to this particular example, but we will consider the general situation in which the scalar field trajectory bends in a smooth way during the phase of kinetic energy domination following an ekpyrotic phase.

In general, in the presence of N scalar fields with general Kähler metric $g_{ij}(\phi)$ on scalar field space, the equation for the evolution of the curvature pertur-

bation on large scales can be rewritten as [61]

$$\dot{\zeta} = \frac{H}{\dot{H}} g_{ij} \frac{D^2 \phi^i}{Dt^2} s^j, \quad (122)$$

where the $N - 1$ entropy perturbations

$$s^i = \delta\phi^i - \dot{\phi}^i \frac{g_{jk}(\phi) \dot{\phi}^j \delta\phi^k}{g_{lm}(\phi) \dot{\phi}^l \dot{\phi}^m} \quad (123)$$

are just the components of $\delta\phi^i$ orthogonal to the background trajectory, and the operator D^2/Dt^2 is the geodesic operator on scalar field space. Here, things simplify because the scalar field space is flat, so the metric is $g_{ij} = \delta_{ij}$, and D/Dt reduces to an ordinary time derivative. Considering only two scalar fields, we have

$$s^1 = -\dot{\phi}_2 \delta s / \sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}, \quad s^2 = +\dot{\phi}_1 \delta s / \sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}. \quad (124)$$

As a first approximation, we will assume that the scalar field trajectory simply reflects off the boundary at $\phi_2 = 0$. The scalar field trajectory is $\dot{\phi}_2 = -\tilde{\gamma}\dot{\phi}_1$, for $t < t_b$, and $\dot{\phi}_2 = \tilde{\gamma}\dot{\phi}_1$, for $t > t_b$, with $\dot{\phi}_1$ constant and negative in the vicinity of the bounce. The bounce leads to a delta function on the right-hand side of (122),

$$\frac{D^2 \phi_2}{Dt^2} = \delta(t - t_{ref}) 2\dot{\phi}_2(t_{ref}^+), \quad (125)$$

where from the higher-dimensional point of view t_{ref} is the time of the bounce of the negative-tension brane. As can be readily seen from (122), if the entropy perturbations already have acquired a scale-invariant spectrum by the time t_{ref} , then the bounce leads to their instantaneous conversion into curvature perturbations with precisely the same long wavelength spectrum. We can estimate the amplitude of the resulting curvature perturbation by integrating equation (123) using (125). Since we have assumed the universe is kinetic-dominated at this time, $H = 1/(3t)$. As pointed out earlier, since the entropy perturbation (95) is canonically normalized, its spectrum is given by (55) up to non-scale-invariant corrections. This expression only holds as long as the ekpyrotic behavior is still underway: the ekpyrotic phase ends at a time t_{end} approximately given by $|V_{end}| = 2/(c^2 t_{end}^2)$. After t_{end} , during the kinetic phase, the entropy perturbation obeys $\ddot{\delta s} + t^{-1} \dot{\delta s} = 0$, which has the solution $\delta s = A + B \ln(-t)$. Matching this solution to the growing mode solution t^{-1} in the ekpyrotic phase, one finds that by t_{ref} the entropy grows by an additional factor of $1 + \ln(t_{end}/t_{ref})$. Employing the Friedmann equation to relate $\dot{\phi}_2 = \tilde{\gamma}\dot{\phi}_1$ to H , putting everything together and restoring both the Planck constant and the Planck mass, we find for the variance of the spatial curvature perturbation in the scale-invariant case,

$$\langle \zeta^2 \rangle = \hbar \frac{c_1^2 |V_{end}|}{3\pi^2 M_{Pl}^2} \frac{\tilde{\gamma}^2}{(1 + \tilde{\gamma}^2)^2} (1 + \ln(t_{end}/t_{ref}))^2 \int \frac{dk}{k} \equiv \int \frac{dk}{k} \Delta_\zeta^2(k) \quad (126)$$

for the perfectly scale-invariant case. Notice that the result depends only logarithmically on t_{ref} : the main dependence is on the minimum value of the effective potential and the parameter c_1 . Observations on the current Hubble horizon indicate $\Delta_\zeta^2(k_0) \approx 2.4 \times 10^{-9}$ with $k_0 = 0.002 \text{ Mpc}^{-1}$ [52]. Ignoring the logarithm in (126), this requires

$$c_1 \sqrt{|V_{end}|} \approx 10^{-3} M_{Pl}, \quad (127)$$

or approximately the GUT scale. This is of course entirely consistent with the heterotic M-theory setting [8]. Note also that this means that the ekpyrotic phase ends at a time of about 10^3 Planck times before the big crunch.

In fact, our approximation of a perfect reflection should really be improved upon, especially since eventually we want to consider second-order corrections. Specific examples [67] show that in the presence of matter on the branes, one expects a smooth repulsive potential and a gradual bending of the trajectory. We will discuss this in more detail shortly in section 4.1.3. It does turn out however that for gradual reflections, and taking into account the evolution of the entropy perturbation during the process of conversion, the efficiency of conversion is only reduced by a factor of about 5 or so [65], and hence the above rough estimate for the amplitude of the perturbations is rather good. Here we simply note that because of the evolution of the entropy perturbation during the time of conversion, sharper reflections in fact lead to less efficient conversions. Turning this the other way around, one can also say that sharper conversions require steeper ekpyrotic potentials (larger values of c_1) to yield an amplitude of cosmological fluctuations compatible with observations.

Conversion during the ekpyrotic phase

In the “new ekpyrotic” models [22,30,57], typically the conversion of entropy to curvature perturbations is assumed to take place because the background trajectory switches from the two-field unstable scaling solution to a late-time single-field attractor solution; in other words, the trajectory starts out close to the ridge of the two-field potential and then falls off one of the steep sides, either because of the initial conditions or due to an additional feature in the potential (for example, one field could see a sharp rise in its potential if it enters the ghost condensate phase before the second one). Adding a feature to the potential makes little difference to the results as long as the reflection remains gradual [65], so in fact we will not consider an additional potential here. Compared to conversion during the kinetic phase, the entropy perturbation falls off by rather less (see section 4.1.3), but the value of $\epsilon_c = \epsilon_{ek}$ is larger. Altogether this implies from (121) that the resulting amplitude of curvature perturbations obeys roughly the same constraints as those discussed above, see equation (127).

Conversion via modulated preheating

Finally, it has been proposed [9] that, instead of converting entropy perturbations into curvature perturbations before the big crunch/big bang transition, the conversion could occur during the phase shortly following the bang through modulated reheating. The concept is that massive matter fields are produced copiously at the brane collision and dominate the energy density immediately after the bang. The massive fields are assumed to couple to ordinary matter with a strength proportional to $h(\delta s)$, so that their decay into ordinary matter occurs at slightly different times depending on the value of δs . In this way, the ordinary matter perturbations inherit the entropic perturbation spectrum. Since the conversion happens while $\epsilon_c \approx 3$, the resulting amplitude of the perturbations should be of the same magnitude as for conversion during the kinetic phase. Note however that a detailed prediction is made difficult by the fact that h is an unknown function of δs . Indeed, the entropy perturbations are converted with an efficiency [9]

$$e = \frac{3}{2} \frac{|h_{,s}|}{h}. \quad (128)$$

In the absence of a concrete model predicting the shape of the function h , it is therefore impossible to go beyond the present order-of-magnitude estimate.

Comparing Predictions for the Spectral Index

Now that we have seen how curvature perturbations with the right amplitude can be generated from a multiple-field ekpyrotic phase, we should add some comments about the spectral index of the perturbations, since this is one of the quantities that can be measured with great precision. As seen above, the curvature perturbations inherit their spectral index from the entropy perturbations, and therefore it is given by (119). In this section, we analyze this relation using several techniques and compare the prediction to those for curvature perturbations in inflation and for a cyclic model in which the single-field time-delay fluctuations are converted to curvature perturbations before/at the bang [61].

As a first approach, let us consider the model-independent estimating procedure used in Ref. [50]. We begin by re-expressing Eq. (119) in terms of \mathcal{N} , the number of e-folds before the end of the ekpyrotic phase (where $d\mathcal{N} = (\epsilon - 1)dN$ and $\epsilon \gg 1$):

$$n_s - 1 = \frac{2}{\epsilon} - \frac{d \ln \epsilon}{d\mathcal{N}}. \quad (129)$$

This expression is identical to the case of the Newtonian potential perturbations derived in [50], except that the first term has the opposite sign. In this expression, $\epsilon(\mathcal{N})$ measures the equation of state during the ekpyrotic phase, which decreases from a value much greater than unity to a value of order unity

in the last \mathcal{N} e-folds. If we estimate $\epsilon \approx \mathcal{N}^\alpha$, then the spectral tilt is

$$n_s - 1 \approx \frac{2}{\mathcal{N}^\alpha} - \frac{\alpha}{\mathcal{N}}. \quad (130)$$

Here we see that the sign of the tilt is sensitive to α . For nearly exponential potentials ($\alpha \approx 1$), the spectral tilt is $n_s \approx 1 + 1/\mathcal{N} \approx 1.02$, slightly blue, because the first term dominates. However, there are well-motivated examples (see below) in which the equation of state does not decrease linearly with \mathcal{N} . We have introduced α to parameterize these cases. If $\alpha > 1.14$, the spectral tilt is red. For example, $n_s = 0.97$ for $\alpha \approx 2$. These examples represent the range that can be achieved for the entropically-induced curvature perturbations in the simplest models, roughly

$$0.97 < n_s < 1.02. \quad (131)$$

For comparison, if we use the same estimating procedure for the Newtonian potential fluctuations in the cyclic model (assuming they converted to curvature fluctuations before the bounce), we obtain $0.95 < n_s < 0.97$. This range agrees with the estimate obtained by an independent analysis based on studying inflaton potentials directly [19]. Furthermore, as shown in Ref. [50], the same range is obtained for time-delay (Newtonian potential) perturbations in the cyclic model, due to the duality in the linear perturbation equations discussed above [18]. Hence, all estimates are consistent with one another, and we can conclude that the range of spectral tilt obtained from entropically-induced curvature perturbations is typically bluer by a few percent.

We note that it is possible that curvature perturbations are created both by the entropic mechanism and by converting Newtonian potential perturbations into curvature perturbations through higher-dimensional effects. In this case, the cosmologically relevant contribution is the one with the bigger amplitude. In particular, the conversion of Newtonian potential perturbations is sensitive to the brane collision velocity [99,78], whereas the entropic mechanism is not. So, conceivably, either contribution could dominate.

A second way of analyzing the spectral tilt is to assume a form for the scalar field potential. Consider the case where the two fields have steep potentials that can be modeled as $V(\phi_1) = -V_0 e^{-\int c d\phi}$ and $\dot{\phi}_2 = \gamma \dot{\phi}_1$. Then Eq. (119) becomes

$$n_s - 1 = \frac{4(1 + \gamma^2)}{c^2 M_{Pl}^2} - \frac{4c_{,\phi}}{c^2}, \quad (132)$$

where we have used the fact that $c(\phi)$ has the dimensions of inverse mass and restored the factors of Planck mass. The presence of M_{Pl} clearly indicates that the first term on the right is a gravitational term. It is also the piece that makes a blue contribution to the spectral tilt. The second term is a non-gravitational term. For a pure exponential potential, which has $c_{,\phi} = 0$, the

non-gravitational contribution is zero, and the spectrum is slightly blue, as our model-independent analysis suggested. For plausible values of $c = 20$ and $\gamma = 1/2$, say, the gravitational piece is about one percent and the spectral tilt is $n_s \approx 1.01$, also consistent with our earlier estimate. However, this case with $c_{,\phi}$ precisely equal to zero is unrealistic. In the cyclic model, for example, the steepness of the potential must decrease as the field rolls downhill in order that the ekpyrotic phase comes to an end, which corresponds to $c_{,\phi} > 0$. If $c(\phi)$ changes from some initial value $\bar{c} \gg 1$ to some value of order unity at the end of the ekpyrotic phase after ϕ changes by an amount $\Delta\phi$, then $c_{,\phi} \sim \bar{c}/\Delta\phi$. When c is large, the non-gravitational term in Eq. (132) typically dominates and the spectral tilt is a few per cent towards the red. For example, suppose $c \propto \phi^\beta$ and $\int c(\phi) d\phi \approx 125$; then, the spectral tilt is

$$n_s - 1 = -0.03 \frac{\beta}{1 + \beta}, \quad (133)$$

which corresponds to $0.97 < n_s < 1$ for positive $0 < \beta < \infty$, in agreement with our earlier estimate.

Finally, we can estimate the running of the spectral index, given by $dn_s/d \ln k$ [54]. Again, we can use the model-independent estimating technique used above. Then, starting with equation (130) and using $\mathcal{N} = \ln(a_{end}H_{end}/aH)$ as well as $k = aH$, we obtain

$$\frac{dn_s}{d \ln k} = -\frac{dn_s}{d\mathcal{N}} = \frac{2\alpha}{\mathcal{N}^{\alpha+1}} - \frac{\alpha}{\mathcal{N}^2}. \quad (134)$$

Since we have $\alpha > 1$, we can see that the running of the spectral index is of order $\mathcal{O}(\mathcal{N}^{-2})$, which is typically $\approx 10^{-3}$ or less. The current WMAP bounds on the running are $|dn_s/d \ln k| \lesssim 0.03$ [52] and are thus easily satisfied.

4.1.3 Non-Gaussianity

Cosmological measurements, as performed for example by the WMAP satellite, are just now becoming sensitive enough to probe the second order corrections to the background evolution of the universe. If we stop at linear order in perturbation theory, then the equations of motion can be derived from a quadratic action S . Hence, the fluctuation modes obey gaussian statistics (as the probability $\sim e^{-S}$). In this sense the second order corrections give us a measure of non-gaussianity in the distribution of cosmological perturbations, and, in conjunction with precision measurements of the spectral index, measures of non-gaussianity provide very sensitive tests of cosmological models. As we will see, ekpyrotic models predict a substantial amount of non-gaussianity, in contrast to simple single-field inflationary models. This is principally due to the fact that the potential is steep, and hence self-interactions of the ekpyrotic field are important, whereas in inflation, the potential is very flat and

self-interactions of the inflaton are negligible. The fact that non-gaussianity will be measured with high precision in the near future (with the upcoming Planck satellite) and that hints of it have already been seen [110] justifies a detailed examination of the ekpyrotic predictions. For conversion during the ekpyrotic phase, the order-of-magnitude of non-gaussianity has been estimated in [30,23] and more detailed calculations were performed in [55,24]; for conversion during the kinetic phase, the non-gaussianity was calculated in [64]. The various models have been compared in [65].

What we have to do is rather clear: namely, we have to repeat the analysis of the previous paragraphs, but this time we need to include all the second order terms as well. To this effect, we will decompose the entropy perturbation into a linear, gaussian part and a second-order perturbation by writing $\delta s = \delta s^{(1)} + \delta s^{(2)}$. Its equation of motion, on large scales and up to second order in field perturbations is then given by [58]

$$\begin{aligned} & \ddot{\delta s} + 3H\dot{\delta s} + (V_{ss} + 3\dot{\theta}^2) \delta s \\ & + \frac{\dot{\theta}}{\dot{\sigma}} (\dot{\delta s}^{(1)})^2 + \frac{2}{\dot{\sigma}} \left(\ddot{\theta} + \dot{\theta} \frac{V_{\sigma}}{\dot{\sigma}} - \frac{3}{2} H \dot{\theta} \right) \delta s^{(1)} \dot{\delta s}^{(1)} \\ & + \left(\frac{1}{2} V_{sss} - \frac{5\dot{\theta}}{\dot{\sigma}} V_{ss} - \frac{9\dot{\theta}^3}{\dot{\sigma}} \right) (\delta s^{(1)})^2 + \frac{2\dot{\theta}}{\dot{\sigma}} \delta \epsilon^{(2)} = 0. \end{aligned} \quad (135)$$

The last term in equation (135) is a non-local term proportional to the difference in spatial gradients between the linear entropy perturbation and its time derivative. This difference evolves as a^{-3} [58], so that it remains approximately constant during the ekpyrotic phase when a is very slowly varying. This ends up being exponentially suppressed compared to the entropy perturbation itself, which grows by a factor of 10^{30} or more during this same period [61]. After the ekpyrotic phase has ended, the non-local term grows because $a \propto (-t)^{-1/3}$, but this growth is negligible compared to the exponential suppression during the ekpyrotic phase. Hence, the non-local term can be safely neglected. Note that this is a closed equation for the entropy perturbation. Here V_{σ} denotes a derivative of the potential along the background trajectory.

It is useful to recast the evolution in terms of the adiabatic and entropic variables σ and s . Up to unimportant additive constants (that we will fix below), they can be defined by

$$\sigma \equiv \frac{\dot{\phi}_1 \phi_1 + \dot{\phi}_2 \phi_2}{\dot{\sigma}} \quad (136)$$

$$s \equiv \frac{\dot{\phi}_1 \phi_2 - \dot{\phi}_2 \phi_1}{\dot{\sigma}}. \quad (137)$$

Then we can expand the potential up to third order as follows [23]:

$$V_{ek} = -V_0 e^{\sqrt{2\epsilon}\sigma} [1 + \epsilon s^2 + \frac{\kappa_3}{3!} \epsilon^{3/2} s^3], \quad (138)$$

where κ_3 is of $\mathcal{O}(1)$ for typical potentials (the case of exact exponentials corresponds to $\kappa_3 = -4\sqrt{2/3}$). The scaling solution (20) can be rewritten as

$$a(t) = (-t)^{1/\epsilon} \quad \sigma = -\sqrt{\frac{2}{\epsilon}} \ln \left(-\sqrt{\epsilon V_0 t} \right) \quad s = 0. \quad (139)$$

The intrinsic non-gaussianity in the entropy perturbation is produced during the ekpyrotic phase and can be determined from the equation of motion for the entropy field (135), which reduces to

$$\ddot{\delta s} + 3H\dot{\delta s} + V_{ss}\delta s + \frac{1}{2}V_{sss}(\delta s^{(1)})^2 = 0. \quad (140)$$

The last term is

$$\frac{1}{2}V_{sss} = -\frac{\kappa_3}{2t^2}\sqrt{\epsilon} \quad (141)$$

and thus the solution, at long wavelengths and up to second order in field perturbations, is given by [55,64]

$$\begin{aligned} \delta s(t) &= \delta s^{(1)}(t) + \delta s^{(2)}(t) \\ &= \delta s_{end} \frac{t_{end}}{t} + \tilde{c} \left(\delta s_{end} \frac{t_{end}}{t} \right)^2, \end{aligned} \quad (142)$$

where

$$\tilde{c} = \frac{\kappa_3 \sqrt{\epsilon}}{8}. \quad (143)$$

Thus, the intrinsic non-gaussianity present in ekpyrotic models (second term in (142) above) is of order $\mathcal{O}(\sqrt{\epsilon_{ek}})$ and it is due to the steepness of the potentials and the resulting self-interactions of the scalar fields. This is in sharp contrast with inflation, where the intrinsic non-gaussianity is extremely small due to the flatness of the potential (or, equivalently, $\epsilon_{inf} \ll 1$).

What is measured is not directly the non-gaussianity present in the entropy perturbation, but the non-gaussianity it imprints on the curvature perturbation. Thus, it is important to know the strength with which this intrinsic non-gaussianity gets transferred to the curvature perturbation. The time evolution of the curvature perturbation to second order in field perturbations and at long wavelengths is given in FRW time by [58]

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\dot{\theta}\delta s + \frac{H}{\dot{\sigma}^2}[(V_{ss} + 4\dot{\theta}^2)(\delta s^{(1)})^2 - \frac{V_{,\sigma}}{\dot{\sigma}}\delta s\dot{\delta s}], \quad (144)$$

where we have omitted a non-local term which can be neglected in ekpyrotic models for the same reason as the non-local term omitted from (135) above. As we saw in equation (121), at linear order, a non-zero entropy perturbation combined with a bending ($\dot{\theta} \neq 0$) of the background trajectory sources the curvature perturbation on large scales and results in a linear, gaussian curvature perturbation $\zeta_L = \int_{\Delta t} \sqrt{\frac{2}{\epsilon_c}} \dot{\theta} \delta s^{(1)}$. Thus the strength of conversion is proportional to $1/\sqrt{\epsilon_c}$ and this dependence on the equation of state significantly influences the magnitude of the second order correction as well, as we will see presently.

As in most inflationary models, the fluctuations are generated by scalar fields with canonical kinetic energy density, so they generate non-gaussianity of the ‘‘local’’ type, as considered in Refs. [106,53,75,7]. The local wavelength-independent non-gaussian contribution to ζ can then be characterized in terms of the leading linear, gaussian curvature perturbation ζ_L according to

$$\zeta = \zeta_L + \frac{3}{5} f_{NL} \zeta_L^2, \quad (145)$$

using the sign convention for wavelength-independent non-gaussianity parameter f_{NL} in [53]. Current observational constraints on f_{NL} have been reported by the WMAP team [52] to lie in the range

$$-9 < f_{NL} < 111 \quad (146)$$

at the 2σ level.

For the two-field models that we are interested in here, Eq. (144) implies that the contributions to f_{NL} can be divided into three parts [64]

$$f_{NL}^{intrinsic} = -\frac{5}{3\zeta_L^2} \int_{\Delta t} \frac{2H}{\dot{\sigma}} \dot{\theta} \delta s^{(2)} \quad (147)$$

$$f_{NL}^{reflection} = \frac{5}{3\zeta_L^2} \int_{\Delta t} \frac{H}{\dot{\sigma}^2} [(V_{ss} + 4\dot{\theta}^2)(\delta s^{(1)})^2 - \frac{V_{,\sigma}}{\dot{\sigma}} \delta s \dot{\delta s}] \quad (148)$$

$$f_{NL}^{integrated} = \frac{5}{6\zeta_L^2} (\delta s^{(1)}(t_{end}))^2, \quad (149)$$

with the total f_{NL} being the sum of all three contributions. $f_{NL}^{intrinsic}$ arises from the direct translation of the intrinsic non-linearity present in the entropy perturbation into a corresponding non-linearity in the curvature perturbation. By contrast, $f_{NL}^{reflection}$ and $f_{NL}^{integrated}$ would be non-zero even if the entropy perturbation were exactly gaussian. Both contributions are due to the non-linear relationship between the curvature perturbation and the entropy perturbation, as expressed in equation (144) - the difference is that $f_{NL}^{integrated}$ gets generated during the ekpyrotic phase and $f_{NL}^{reflection}$ during the conversion.

The intrinsic non-gaussianity can be estimated by combining equations (121), (142), (143) and (147):

$$\begin{aligned}
f_{NL}^{intrinsic} &\approx \pm \frac{5}{3\zeta_L^2} \int_{\Delta t} \sqrt{\frac{2}{\epsilon_c}} \dot{\theta} \tilde{c} \delta_S^{(1)2} \\
&\sim \sqrt{\epsilon_c \epsilon_{ek}}.
\end{aligned}
\tag{150}$$

It is given by the geometric mean of the ϵ parameters during the phases of generation and conversion of the perturbations. This is perhaps the most important result because it fixes a rough magnitude for $|f_{NL}|$ which can only be significantly reduced through accidental cancelations due to the other terms or other effects not included in the present analysis. It explains in a nutshell why the non-gaussianity in ekpyrotic/cyclic models is necessarily more than an order of magnitude greater than in simple inflation models, and it explains why the equation of state during conversion can have a significant quantitative effect on the prediction.

To go beyond this qualitative estimate to a more precise one, we need to take account of the details of the conversion mechanism. We will again discuss the various possibilities considered above in turn.

Conversion during kinetic energy domination

We can immediately perform an order-of-magnitude estimate of the non-gaussianity generated by this conversion mechanism (described in section 4.1.2) since during the kinetic phase $\epsilon_c = 3$, and so (150) would lead us to expect f_{NL} to be of order

$$f_{NL} \sim \sqrt{\epsilon_{ek}} \sim \mathcal{O}(c_1). \tag{151}$$

We will treat the bend as if only one field reflects (ϕ_2), and we will consider the case where $\dot{\theta} > 0$, see Fig. 6. Other cases can be related to these representative examples by changing the coordinate system in field space appropriately. In the heterotic M-theory example, the reflection occurs because ϕ_2 comes close to a boundary of moduli space ($\phi_2 = 0$) and is forced to bounce [67]. For the purposes of this study, we will treat the reflection as being due to a potential, $V^R(\phi_2)$, an additional contribution unrelated to the exponential potentials that were dominant during the ekpyrotic phase (but which are negligible during the kinetic energy dominated phase).

It is important to know the evolution of the entropy perturbation during the process of conversion. If there is a phase of pure kinetic energy domination before the conversion, then the background scalar field trajectory is also a straight line during this phase, but with the potentials being irrelevant. The

equation of motion reduces to

$$\ddot{\delta s} + \frac{1}{t}\dot{\delta s} = 0, \quad (152)$$

and by matching onto the ekpyrotic solution and its first time derivative at $t = t_{end}$ we find

$$\begin{aligned} \delta s(t) &= \delta s^{(1)}(t) + \delta s^{(2)}(t) \\ &= \delta s_{end} \left(1 + \ln \frac{t_{end}}{t}\right) + \tilde{c} \delta s_{end}^2 \left(1 + 2 \ln \frac{t_{end}}{t}\right). \end{aligned} \quad (153)$$

Incidentally, note that

$$t \frac{\dot{\delta s}}{\delta s} \sim \frac{1}{\ln(-t)} \quad (154)$$

during the kinetic phase, while during the ekpyrotic phase $t\dot{\delta s} \sim \delta s$. This observation will simplify our analysis later on.

During the conversion, even though the kinetic energy of the scalar fields is still the dominant contribution to the total energy, the potential $V^R(\phi_2)$ that causes the bending has a significant influence on the evolution of the entropy perturbation. To analyze this, we will approximate (as in [24]) the bending of the trajectory to be gradual by taking $\dot{\theta}$ constant and non-zero for a period of time Δt , starting from $t = t_{ref}$. Note that, assuming the total angle of bending is $\mathcal{O}(1)$ radian, we have $|\dot{\theta}| \approx 1/|t_{ref}|$ in this case. Then one can relate the derivatives of the potential to expressions involving $\dot{\theta}$, for example

$$V_{ss}^R = -2\dot{\theta}^2 + \frac{\dot{\theta}}{\gamma t}. \quad (155)$$

Assuming a gradual conversion ($\Delta t \sim t_{ref}$), we can ignore higher derivatives of θ . (In [64] it was shown that sharp transitions lead to unacceptably large values of f_{NL} ; hence these cases are of less phenomenological interest.) To satisfy the constraint on the amplitude of the curvature perturbation obtained by the WMAP observations, we set $t_{ref} \approx -10^3 M_{Pl}^{-1}$ [61]. At linear order, the equation of motion (135) then reads

$$\ddot{\delta s}^{(1)} + 3H\dot{\delta s}^{(1)} + \left(\dot{\theta}^2 + \frac{1}{\gamma t}\dot{\theta}\right)\delta s^{(1)} = 0, \quad (156)$$

As indicated by equation (154), we can set $\dot{\delta s}^{(1)} = 0$ as a first approximation and, thus, neglect the damping term in the equation of motion. Also, we will simply evaluate the coefficient of the last term midway through the reflection, and define

$$\omega \equiv \dot{\theta} \sqrt{1 + \frac{1}{\dot{\theta}\gamma(t_{ref} + \Delta t/2)}}. \quad (157)$$

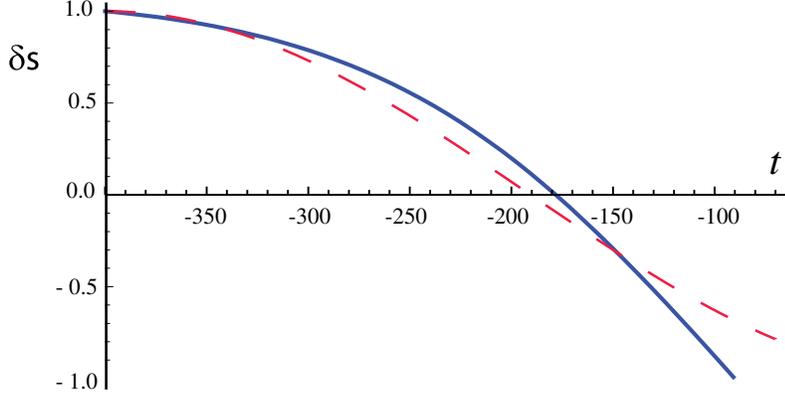


Fig. 7. The evolution of the linear entropy perturbation during conversion: the solid (blue) line shows the actual evolution calculated numerically, while the dashed (purple) line shows the approximate solution (158) with $\omega = 3$, $\dot{\theta} = -1/t_{ref}$ and $t_{ref} = -400M_{Pl}^{-1}$. For the purposes of illustration, $\delta s^{(1)}(t_{ref})$ has been normalized to 1.

For a gradual reflection $\omega \approx (2 - 3)\dot{\theta}$. Then the solution for the linear entropy perturbation is

$$\delta s^{(1)} = \delta s^{(1)}(t_{ref}) \cos \omega(t - t_{ref}). \quad (158)$$

Instead of continuing to grow logarithmically, the entropy perturbation actually falls off during the conversion, see Fig. 7. This has the consequence that the conversion from entropy to curvature perturbations is less efficient than one might have naively thought. From (121), we can estimate

$$\zeta_L = \sqrt{\frac{2}{3}} \dot{\theta} \int_{\Delta t} \delta s^{(1)} = \sqrt{\frac{2}{3}} \frac{\dot{\theta}}{\omega} \delta s^{(1)}(t_{ref}) \sin \omega \Delta t, \quad (159)$$

where we have used $\epsilon_c \approx 3$, which is a good approximation for subdominant reflections (and acceptable for the estimating purposes for dominant ones). Since $\omega \Delta t \approx 3$, the entropy perturbation evolves over nearly a half-cycle and consequently $\sin \omega \Delta t$ is a small factor which we will take to be about 1/3 in our estimates, which fits well with numerical results.

We also need to know the evolution of the second order entropy perturbation during the time of conversion. Using the same approximations as above (implying that we can neglect V_{sss} compared to $\dot{\theta}V_{ss}/\dot{\sigma}$ and $\dot{\theta}^3/\dot{\sigma}$ by the time the conversion is underway), the equation of motion (135) simplifies to

$$\ddot{\delta s}^{(2)} + \omega^2 \delta s^{(2)} - \frac{\dot{\theta}}{\dot{\sigma}} \omega^2 (\delta s^{(1)})^2 = 0, \quad (160)$$

where $\delta s^{(1)}$ is given in (158). Putting $\dot{\theta} \approx \dot{\sigma}$ at the start of the reflection and keeping in mind that we impose the boundary condition $\dot{\delta s}^{(2)} \approx 0$, the solution for the second order entropy perturbation is given by

$$\begin{aligned}
\delta s^{(2)} = & \delta s^{(2)}(t_{ref}) \cos[\omega(t - t_{ref})] \\
& + \frac{1}{12} (\delta s^{(1)}(t_{ref}))^2 \left(-4 \cos[\omega(t - t_{ref})] \right. \\
& + 4 \cos^4[\omega(t - t_{ref})] + 9 \sin^2[\omega(t - t_{ref})] \\
& \left. + \sin[\omega(t - t_{ref})] \sin[3\omega(t - t_{ref})] \right). \tag{161}
\end{aligned}$$

At large ϵ_{ek} (or, equivalently, large $|\tilde{c}|$), the second order entropy perturbation falls off in the same way as the linear perturbation, but at small $|\tilde{c}|$ there are significant corrections to this behavior. We will also need the integral

$$\begin{aligned}
C^{-2} \int_{\Delta t} \delta s^{(2)} = & \frac{(1 + 2 \ln(t_{end}/t_{ref})) \sin(\omega \Delta t)}{(1 + \ln(t_{end}/t_{ref}))^2 \omega} \tilde{c} \\
& + \frac{\Delta t}{2} - \frac{\sin(\omega \Delta t)}{3\omega} - \frac{\sin(2\omega \Delta t)}{12\omega}, \tag{162}
\end{aligned}$$

where $C = \delta s^{(1)}(t_{ref})$. In all cases of interest, the last two terms are negligible.

We are finally in a position to evaluate the various contributions to the non-linearity parameter f_{NL} . The intrinsic contribution, defined in (147), becomes

$$f_{NL}^{intrinsic} \approx A \kappa_3 \sqrt{\epsilon_{ek}} + B, \tag{163}$$

with

$$\begin{aligned}
A = & \frac{5\omega(1 + 2 \ln(t_{end}/t_{ref}))}{8\sqrt{6}\dot{\theta}(1 + \ln(t_{end}/t_{ref}))^2 \sin(\omega \Delta t)} \\
B = & \frac{5\omega^2}{2\sqrt{6}\dot{\theta}^2 \sin^2(\omega \Delta t)}. \tag{164}
\end{aligned}$$

Eq. (163) is one of our key results because it shows that the essential contribution scales in a simple way with ϵ_{ek} and has a value that exceeds the total f_{NL} for simple inflationary models by more than an order of magnitude. As suggested by Eq. (150), the first term in $f_{NL}^{intrinsic}$ can be re-expressed as $\mathcal{O}(\sqrt{\epsilon_c \epsilon_{ek}})$ with $\epsilon_c = 3$. This contribution comes directly from the non-zero $\delta s^{(2)}$ generated during the ekpyrotic phase, which is due to the third derivative w.r.t. s of the ekpyrotic potential. It is interesting to note that, in the case where $V_{sss} = 0$ or $\kappa_3 = 0$, $f_{NL}^{intrinsic}$ is nevertheless non-negligible because of the positive offset B generated by the linear entropy perturbation $\delta s^{(1)}$ during conversion, the piece proportional to $(\delta s^{(1)})^2$ in (161).

Using the definition (148) together with (155) it is straightforward to see that for conversion during the kinetic phase, $f_{NL}^{reflection}$ is always negative, and it can be estimated as

$$\begin{aligned}
f_{NL}^{reflection} &\approx \frac{5}{6\zeta_L^2} \int_{\Delta t} (2\dot{\theta}^2 t + \frac{\dot{\theta}}{\gamma})(\delta s^{(1)})^2 \\
&\approx -\frac{15\omega^2}{8\dot{\theta}^2 \sin^2(\omega\Delta t)} \frac{|t_{ref} + \Delta t/2|}{\Delta t},
\end{aligned} \tag{165}$$

where we have used $\gamma t_{ref} \dot{\theta} \approx 1$ and

$$\int_{\Delta t} t \sin^2 \omega(t - t_{ref}) \approx \frac{1}{2} (t_{ref} + \frac{\Delta t}{2}) \Delta t, \tag{166}$$

which is a valid approximation since $\delta s^{(1)}$ evolves over approximately a half-cycle. We have neglected the contribution due to the term proportional to $ds ds$, since it is suppressed on account of (154). The integrated contribution to f_{NL} generated during the ekpyrotic phase gives an additional contribution of

$$\begin{aligned}
f_{NL}^{integrated} &= \frac{5}{12\zeta_L^2} (\delta s^{(1)}(t_{end}))^2 \\
&\approx \frac{5\omega^2}{8\dot{\theta}^2 (1 + \ln(t_{end}/t_{ref}))^2 \sin^2(\omega\Delta t)}.
\end{aligned} \tag{167}$$

Note that neither the reflected nor the integrated contributions depend on ϵ_{ek} ; they both simply shift the final result by a number depending on the sharpness of the transition and the duration of the purely kinetic phase respectively.

The total f_{NL} is the sum of all the above contributions. Since we are only considering gradual conversions, we expect the ratio $|t_{ref} + \Delta t/2|/\Delta t$ to lie between 1 and 2. Also, the duration of the pure kinetic phase between the end of the ekpyrotic phase and the conversion is necessarily rather short, so that we expect $\ln(t_{end}/t_{ref}) \lesssim \mathcal{O}(1)$.

Putting everything together, we get the following estimates:

$$f_{NL}^{intrinsic} \approx 10\tilde{c} + 50 \tag{168}$$

$$f_{NL}^{reflection} \approx -50 \tag{169}$$

$$f_{NL}^{integrated} \approx 5. \tag{170}$$

The analytic estimate for $f_{NL}^{reflection}$ suggests a range that could extend to -100 , but the value above is more consistent with the exact numerical calculations for a wide range of parameters. Altogether, we end up with the following fitting formula

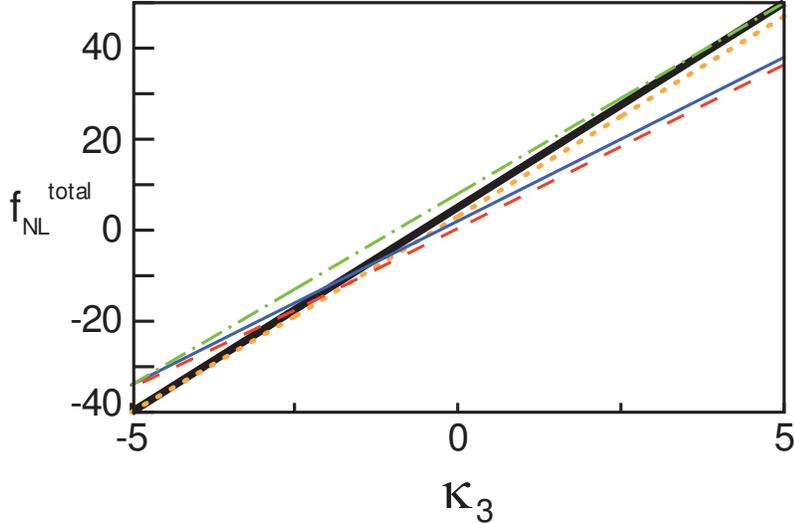


Fig. 8. A comparison of the results from numerical calculations with the fitting formula given in Eq. (172) and indicated by the thick black line. Here we have fixed the value of $\epsilon_{ek} = 36$. The plot confirms that f_{NL} then grows linearly with κ_3 , in good agreement with (172). The sample models (dashed and dotted lines) are representative of the range of models and parameters shown in [64]. We have similarly checked the dependence on $\sqrt{\epsilon}$ when the value of κ_3 is kept fixed.

$$f_{NL}^{total} \approx 10\tilde{c} + 5 \quad (171)$$

$$\approx \frac{3}{2}\kappa_3\sqrt{\epsilon_{ek}} + 5. \quad (172)$$

Fig. 8 shows that this formula agrees well with the results from exact numerical calculations. In particular, we have been able to estimate the slope, and hence the dependence on $\kappa_3\sqrt{\epsilon_{ek}}$, rather accurately.

Conversion during the ekpyrotic phase

If the conversion happens during the ekpyrotic phase, *i.e.* while the ekpyrotic potentials are relevant, $\epsilon_c = \epsilon_{ek}$, and from (150) we expect the non-gaussianity to be of order

$$f_{NL} \sim \epsilon_{ek} \sim \mathcal{O}(c_1^2), \quad (173)$$

i.e. we expect the non-gaussianity to be larger by a factor of c_1 compared to the case of conversion during the kinetic phase.

As shown by Koyama *et al.* [55], the $\delta\mathcal{N}$ formalism is well suited to treating this case, and the derivation surprisingly quick. According to the $\delta\mathcal{N}$ formalism [91,87], the comoving curvature perturbation is identified with a perturbation in the number of e-folds of ekpyrosis (the perturbations are identified on a uniform density hypersurface at $t = t_f$, starting the evolution from a flat

initial hypersurface at $t = t_i$)

$$\zeta = \delta\mathcal{N} = \mathcal{N}_{,\alpha_g}\delta\alpha_g + \frac{1}{2}\mathcal{N}_{,\alpha_g\alpha_g}(\delta\alpha_g)^2, \quad (174)$$

where we have expanded $\delta\mathcal{N}$ up to second order in terms of a gaussian variable α_g . We will specify the meaning of α_g shortly. Then, comparing with the definition (145), we immediately see that the non-gaussianity parameter is given by

$$f_{NL} = \frac{5}{6} \frac{\mathcal{N}_{,\alpha_g\alpha_g}}{(\mathcal{N}_{,\alpha_g})^2}. \quad (175)$$

We can evaluate \mathcal{N} by using the one and two-field versions of the scaling solution (20), with the result that [55]

$$\mathcal{N} = \int_{t_i}^{t_{ref}} H dt + \int_{t_{ref}}^{t_f} H dt = -\frac{2}{c_j^2} \ln |H_{ref}| + constant, \quad (176)$$

where the index j corresponds to the field that becomes frozen in the late-time single-field solution. Here we have assumed that at time t_{ref} the trajectory instantaneously switches from the unstable two-field scaling solution to a single-field attractor scaling solution [57]. Now, using that $\delta s \propto t^{-1} \propto H$, we can rewrite equation (142) as

$$\delta s = \alpha_g H + \tilde{c}(\alpha_g H)^2, \quad (177)$$

whence α_g is seen to be a parameter distinguishing different trajectories. Turning this relation around, we have that

$$\alpha_g = \frac{\delta s}{H}(1 - \tilde{c}\delta s). \quad (178)$$

Hence at the time of transition (reflection) t_{ref} , when $\delta s = \delta s_{ref}$ is fixed, we have that $\alpha_g \propto H_{ref}^{-1}$. Thus, using (176), we can rewrite (174) as

$$\delta\mathcal{N} = \frac{2}{c_j^2\alpha_g}\delta\alpha_g - \frac{1}{c_j^2\alpha_g^2}(\delta\alpha_g)^2. \quad (179)$$

It immediately follows that the non-gaussianity parameter f_{NL} is given by [55]

$$f_{NL} = \frac{5}{6} \left(\frac{-2}{c_j^2\alpha_g^2}\right) \left(\frac{c_j^2\alpha_g}{2}\right)^2 = -\frac{5}{12}c_j^2, \quad (180)$$

where, again, the index j corresponds to the field that becomes frozen in the late-time single-field solution. This expression has been checked numerically, both in [55] and in [65], by different methods, and the numerics have been found to agree well with the analytic formula above. Note the particular feature that the sign of f_{NL} is always positive for these cases.

A qualitative understanding of this result can be achieved in the present context as follows: from (147) we can see that, since $H/\dot{\sigma}$ is negative and approximately constant, the sign of $f_{NL}^{intrinsic}$ is given by the sign of $\int \dot{\theta} \delta s^{(2)}$. In practice, numerical simulations indicate that, for conversion during the ekpyrotic phase, we have

$$|\dot{\theta}| \approx \frac{1}{10|t|} \quad (181)$$

during most of the conversion, but with $\dot{\theta}$ growing faster towards the end. We will use this numerical input to guide our analysis. The sign of $f_{NL}^{intrinsic}$ is essentially determined by the sign of $\dot{\theta} \delta s^{(2)}$ towards the end of the period of conversion. Naively it is difficult to perform a purely analytic estimate of f_{NL} in the current scheme, since all the terms in the equation of motion (135) go as $c_1 t^{-4}$; however, equation (181) tells us that

$$|V_{ss}| = 2/t^2 \gg \dot{\theta}^2, \quad (182)$$

and so there are surprisingly few terms in the equation of motion for $\delta s^{(2)}$ that are actually important during (most of) the time of conversion. In fact, to a first approximation, we are simply left with

$$\delta s^{(2)\ddot{}} = \left(-\frac{1}{2}V_{sss} + \frac{\dot{\theta}}{\dot{\sigma}}V_{ss}\right)(\delta s^{(1)})^2. \quad (183)$$

V_{sss} decreases in importance as the single-field scaling solution is reached. Thus, even though the V_{sss} term determines the initial evolution of $\delta s^{(2)}$, eventually the V_{ss} term dominates. Then, since $V_{ss} < 0$ it is easy to see that the sign of $\delta s^{(2)}$ is always driven to be opposite to that of $\dot{\theta}$, and consequently $f_{NL}^{intrinsic}$ is negative in all cases.

Since $f_{NL}^{reflection}$ is proportional to ζ_L^{-2} , it provides a contribution of the same order $\mathcal{O}(\epsilon_{ek})$. Using (182), it is straightforward to see that the part of $f_{NL}^{reflection}$ that is proportional to $(ds)^2$ is always positive, while the part proportional to $ds ds$ is always negative. This implies that there will be a competition between the two contributions. Numerical integration then shows that the part proportional to $(ds)^2$ approximately cancels out $f_{NL}^{intrinsic}$, while the $ds ds$ part by itself is very close in numerical value to the final answer. In all cases $f_{NL}^{integrated}$ is completely negligible.

Consequently, the total f_{NL} turns out to be negative and moreover in good agreement with the δN result (180) [55], as shown in Table 1. Clearly, the δN formalism provides a fast and elegant derivation of non-gaussianity for conversion during the ekpyrotic phase that agrees well with direct integration of the equations of motion. (Note, however, no analogous δN approach applies to conversion in the kinetic energy dominated phase.)

The conclusion that f_{NL} is substantially less than zero differs from Ref. [24];

c_1	c_2	$f_{NL,\delta N}$	f_{NL}
10	10	-41.67	-39.95
10	15	-41.67	-40.45
10	20	-41.67	-40.62
15	10	-93.75	-91.01
15	15	-93.75	-92.11
15	20	-93.75	-92.49
20	10	-166.7	-162.5
20	15	-166.7	-164.4
20	20	-166.7	-165.1

Table 1

Ekpyrotic conversion: the values of f_{NL} estimated by the dN formalism compared to the numerical results obtained by directly integrating the equations of motion.

their approximation method was incomplete though, since the term proportional to $ds ds$ in (144) was not included. As shown above, this term typically contributes significantly to the final result. Also, the evolution of the entropy perturbation during conversion was neglected, which meant, for example, that values of $f_{NL}^{intrinsic}$ of either sign were obtained. Our results here show that their approximations were too crude, and that f_{NL} is generally negative with $f_{NL} \lesssim -20$. This is inconsistent with current limits by roughly 3σ [52].

Hence, we conclude that models with conversion during the ekpyrotic phase ($\epsilon_c = \epsilon_{ek} \gg 1$) are difficult to accommodate with current observations in contrast to models in which conversion occurs during the kinetic energy dominated phase ($\epsilon_c = 3$).

Conversion after the crunch/bang transition

If the conversion of entropy to curvature perturbations occurs via the scenario of modulated preheating after the big bang [9], then the conversion happens while $\epsilon_c \approx 3$, as described in section 4.1.2. Thus we can expect an intrinsic contribution to f_{NL} of

$$f_{NL} \sim \sqrt{\epsilon_{ek}} \sim \mathcal{O}(c_1), \quad (184)$$

i.e. f_{NL} is of the same order as in the case of conversion during the kinetic phase preceding the big crunch. Moreover, as shown in [9], no significant additional contributions to f_{NL} are expected. Therefore, models of this type are subject to roughly the same observational constraints as the models where the conversion happens during the kinetic phase before the bang and where the intrinsic contribution to the non-gaussianity is dominant. Note however that

the entropy perturbations are converted with an unknown efficiency [9]

$$e = \frac{3}{2} \frac{|h_{,s}|}{h}. \quad (185)$$

Since the non-linearity in the entropy field is of magnitude \tilde{c} , this implies a non-linearity in the curvature perturbation given by

$$f_{NL} \approx \pm \frac{\tilde{c}}{e}, \quad (186)$$

where the \pm sign reflects our ignorance of the sign of $h_{,s}$ and of the direction of bending of the scalar field trajectory. Thus, in the absence of a more detailed model, we cannot go beyond this rough order-of-magnitude estimate at present.

The analysis of non-gaussianity can be summarized in a few rules of thumb:

- The intrinsic contribution to $|f_{NL}|$ is proportional to the geometric mean of ϵ_{ek} and ϵ_c , which is at least two orders of magnitude greater than in simple inflationary models (where $|f_{NL}^{intrinsic}| = \mathcal{O}(0.1)$). When all contributions are considered, the total f_{NL} is generically more than an order of magnitude greater than in simple inflationary models, (where $f_{NL}^{total} = \mathcal{O}(1)$).
- For a fixed value of κ_3 , the value of $|f_{NL}^{total}|$ is correlated with the spectral tilt: smaller $|f_{NL}^{total}|$ implies smaller ϵ_{ek} , which tends to make the spectral tilt bluer. Current limits on f_{NL} fit well with limits on the spectral tilt for the simplest models with conversion during the kinetic energy driven phase before the bang or during reheating after the big bang.
- Models in which the conversion occurs during a phase with larger ϵ_c produce a larger intrinsic $|f_{NL}|$ and are more difficult to fit with the current observed limits on f_{NL} and spectral tilt. In particular, models with conversion in the ekpyrotic phase favor $|f_{NL}|$ large and *negative*.
- Cases in which the intrinsic contribution to f_{NL} is much smaller than the reflection plus integrated contributions produces very large values of $|f_{NL}| \gg 100$ that are inconsistent with current observational bounds. This includes all cases where the conversion is sharp.

The analysis suggests a useful characteristic plot for differentiating cosmological models: f_{NL} versus tilt. Figure 9 illustrates the prediction for a cyclic model in which the conversion from entropic to curvature perturbations occurs in the kinetic energy dominated epoch following the ekpyrotic phase. Here we keep the skewness κ_3 of the potential fixed, as we vary its steepness ϵ . The prediction is a swath whose width is largely due to the uncertainty in the trajectory, parameterized by γ . Although the swath includes f_{NL} near zero, positive f_{NL} between 10 and 100 is preferred for tilts in the range suggested

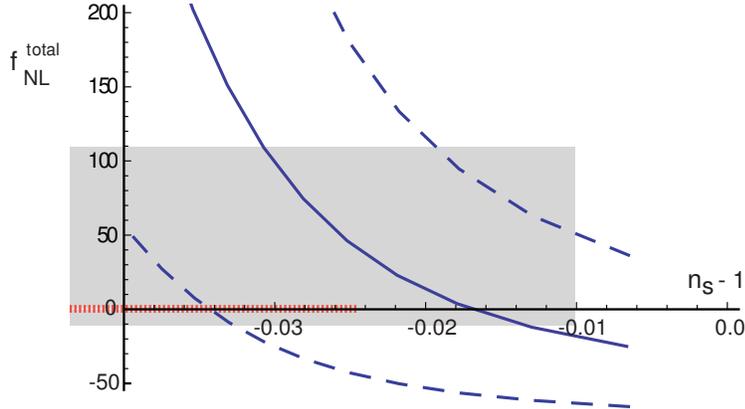


Fig. 9. A plot for characterizing the correlation between $|f_{NL}|$ and scalar spectral tilt, $n_s - 1$, here illustrated for the case of the cyclic model in which the conversion from entropic to curvature perturbations occurs during the kinetic energy dominated phase just before the big crunch/big bang transition. Different curves correspond to different fixed amounts of skewness κ_3 in the potential (the central curve corresponds to $\kappa_3 = 4\sqrt{2/3}$), while we vary the steepness of the potential ϵ_{ek} . The curves show the general trend that $|f_{NL}|$ increases as the spectrum becomes redder. Simple inflationary models correspond to the narrow horizontal hashed (red) strip with $|f_{NL}| \lesssim 1$. The shaded rectangle represents the current observational constraints on f_{NL} and tilt (95% confidence) from WMAP5 [52].

by WMAP5. The prediction for simple inflationary models is confined to the narrow band $\Delta f_{NL} \lesssim 1$ around zero.

The results are surprisingly predictive. If observations of f_{NL} lie in the range predicted by the intrinsic contribution of either inflationary or ekpyrotic/cyclic models, it is reasonable to apply Occam's razor and Bayesian analysis to favor one cosmological model over the other. Combining with measurements of the spectral tilt significantly sharpens the test. The current observational bounds obtained by the WMAP satellite are still inconclusive, but it is clear from the estimates presented here that non-gaussianity should be detected by the Planck satellite if the ekpyrotic/cyclic model is correct. At the same time, this provides a strong incentive to further refine other methods of measuring non-gaussianity, such as looking for evidence in measurements of the large scale structure of the universe [89]. As an example, a positive f_{NL} would mean that the largest structures formed slightly earlier, and thus would have drawn more mass out of the voids, and hence one might for example expect slightly emptier voids, as compared to a perfectly gaussian distribution of primordial density fluctuations.

4.2 Tensor perturbations

The ekpyrotic phase also generates gravitational waves, but with a spectrum radically different from the inflationary prediction. Thus, gravitational waves provide another useful test for differentiating between models of the early universe. Unfortunately, it is unclear at present how soon measurement devices will be sensitive enough to probe the gravitational wave background.

Tensor perturbations are simpler to analyze than scalar perturbations since there are no issues of gauge variance. In the context of ekpyrotic models, the analysis was first performed in [48,17]. Here, we will follow the treatment of [18]. The perturbed metric is given by

$$ds^2/a^2 = -d\tau^2 + [\delta_{ij} + 2h_T Y_{ij}^{(2)}] dx^i dx^j \quad (187)$$

where $Y_{ij}^{(2)}$ is a tensor harmonic. The tensor perturbation h_T is gauge-invariant and obeys the equation of motion

$$h_T'' + 2(a'/a)h_T' + k^2 h_T = 0. \quad (188)$$

It is useful to define the re-scaled variable $f_T \equiv ah_T$ which obeys

$$f_T'' + (k^2 - a''/a)f_T = 0. \quad (189)$$

Again, the standard vacuum choice is the Minkowski vacuum of a comoving observer in the far past, corresponding to the boundary condition

$$f_T \rightarrow \frac{1}{\sqrt{2k}} e^{-ik\tau} \quad \text{as } \tau \rightarrow -\infty. \quad (190)$$

We can now solve for f_T , just as in the scalar case. But it is simpler to notice that (58) implies $z(\tau) \propto a(\tau)$, and hence $z''/z = a''/a$ when ϵ is constant. Thus, since v and f_T obey identical equations of motion (compare (67) with (189)) and boundary conditions (compare (73) with (190)), we find

$$f_T = v = \mathcal{P}_2 \sqrt{\frac{\pi x}{4k}} H_\beta^{(1)}(x). \quad (191)$$

The tensor spectral index is defined in the long-wavelength limit by $k^3 |f_T|^2 \propto k^{n_T}$. Using the small x expansion of the Hankel function and (191) we find

$$n_T = 3 - 2\beta = 3 - \left| \frac{\epsilon - 3}{\epsilon - 1} \right|. \quad (192)$$

Thus, in ekpyrotic models, the tensor spectrum is very blue, $n_T \approx 2$. There is a cut-off of the spectrum at high frequencies [17], due to a combination of

two facts: first, gravitational waves are not confined to the brane - they see the whole spacetime, as they are excitations of the metric. And secondly, after the ekpyrotic phase has ended, the kinetic dominated phase starts on the brane worldvolume. However, as described in section 3.2, the corresponding higher-dimensional spacetime near the brane collision is compactified Milne spacetime, which is locally Minkowski space. Thus, gravitational waves with wavelengths smaller than about the GUT scale, which leave the horizon after the end of the ekpyrotic phase, behave as in Minkowski space and are not amplified (see also section 6 for more details on the higher-dimensional spacetime near the collision). The high-frequency modes just below the cut-off lead to the strongest constraint on the gravitational wave spectrum, as they are susceptible of interfering with big bang nucleosynthesis. The corresponding constraint requires a minimum temperature at reheating, and will be presented in section 5 along with other constraints that ekpyrotic and cyclic models have to satisfy.

The ekpyrotic spectrum is in marked contrast with the model of inflation, which predicts a nearly scale-invariant spectrum of gravitational waves, essentially because there, the gravitational waves are produced in the same way as the scalar perturbations. Because of the very blue spectrum, gravity waves are just about unobservable in ekpyrotic models [48]: if we assume the same Hubble parameter H_b at reheating in a model of inflation, and at the brane collision in an ekpyrotic model, then modes of wavelength H_b^{-1} will have approximately the same amplitude, namely H_b/M_{Pl} . But now consider a mode of wavelength comparable to the present-day horizon, $\sim H_0^{-1}$. Then, for inflation, we would expect an amplitude of similar magnitude due to the approximate scale-invariance of the spectrum. But for ekpyrosis, the current horizon is of the order of 60 e-folds larger than H_b^{-1} , and hence, due to the blue spectrum, the amplitude of a horizon-sized gravity wave today would be exponentially small and thus unobservable. In fact, on large scales, one would expect the leading contribution to the gravitational wave background to be induced by the backreaction on the geometry of the first-order scalar perturbations [11].

Hence gravitational waves provide one of the crucial observational tests that are capable of unambiguously ruling out ekpyrotic models. If gravitational waves are detected, and it can be established that their spectrum is close to scale-invariant, this will constitute very strong evidence in favor of inflation. If, on the other hand, primordial gravitational waves remain unobserved, ekpyrotic models will deserve our attention and we should look for other significant tests (such as non-gaussianity) in order to improve our understanding of the early universe.

5 The End is the Beginning: a Cyclic Model of the Universe

We have seen how the ekpyrotic phase can resolve the standard puzzles of big bang cosmology, and how it can generate scalar and tensor fluctuations with interesting properties. However, this leaves many aspects of our universe unaddressed, and foremost the rather mysterious dark energy. The cyclic model [92–96,51,35] is an ambitious attempt at providing a complete history of the universe, within the framework of a braneworld view of the universe, while incorporating both the ekpyrotic mechanism and dark energy in an essential way. We will first outline the main ideas, then discuss the constraints that the cyclic model has to satisfy and finally describe the global view and new possibilities that the model offers. The cyclic model relies on having a scalar field potential of the general shape depicted in figure 10. The plausibility of such a potential arising within the context of string theory will be discussed in section 6. The idea is that the ekpyrotic phase, which precedes the big crunch/big bang transition, is itself preceded by a phase where the potential is positive and flat. During this phase, the radion field rolls down the potential very slowly, and this in fact provides a period of dark energy domination (in the form of quintessence [84]). But this could be our current universe! The dark energy that we see today is reinterpreted as a small attractive force between our brane and the second, parallel, end-of-the-world brane. At some point in the future, this force becomes stronger; the potential energy becomes negative, and the universe reverts from expansion to contraction. We then enter an ekpyrotic phase, which locally flattens and homogenizes the branes. After a brief phase of kinetic energy domination, the branes collide, matter and radiation are produced and brane dwellers experience a big bang. After the bang, the scalar field acquires a small boost and quickly rolls back across the potential well and onto the positive energy plateau, where the scalar gets quasi-stabilized. Now the universe undergoes the usual periods of radiation and matter domination, while the scalar field starts rolling slowly back. Eventually, the resulting dark energy comes to dominate the energy density of the universe, and the whole cycle starts again. In this way, physical processes in the universe today provide the initial conditions for the next cycle. What should be noted is the amazing economy of ingredients: only branes and their motion are used to address the general evolution of the universe over its entire history! Of course, aesthetic appeal is nice in a physical model, but we must ask the question: can this work? In order to answer this, we must have a closer look at the details.

First, there is the question of entropy [93]. The second law of thermodynamics implies that the entropy of each cycle should be larger than that of the previous cycle; but this seems to preclude the possibility of a truly cyclic universe (and historically, Tolman used this reasoning to show the impossibility of cyclic models in closed universes with zero cosmological constant [101]). The

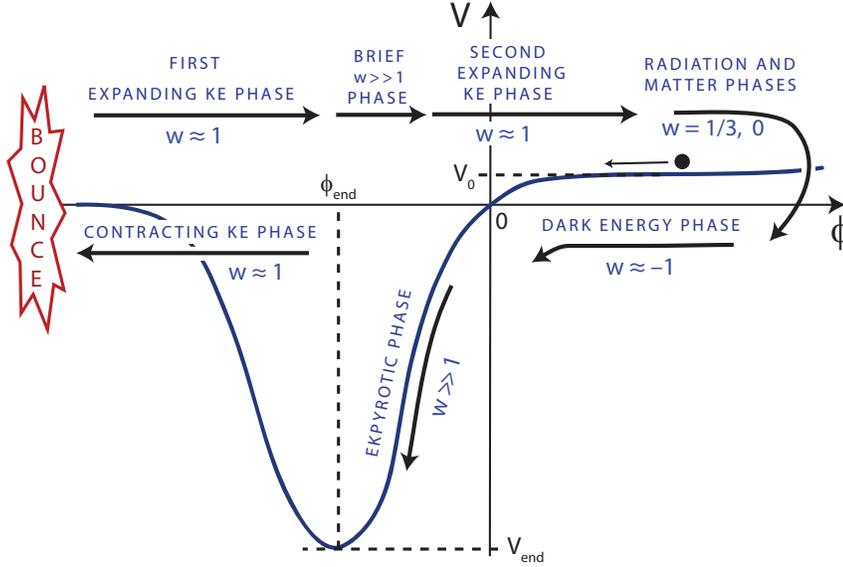


Fig. 10. The potential for the cyclic universe integrates the ekpyrotic part and a quintessence epoch, but is irrelevant at the brane collision. A possible form for the potential is $V(\phi) = V_0(e^{b\phi} - e^{-c\phi})F(\phi)$, with $b \ll 1$, $c \gg 1$ and $F(\phi)$ tends to unity for $\phi > \phi_{end}$ and to zero for $\phi < \phi_{end}$. Reproduced with permission from [35].

resolution is that, as we will see shortly, there is a huge net expansion every cycle, and hence, even though the total entropy always increases, the entropy gets diluted during the dark energy phase, and the entropy density can return to approximately the same value cycle after cycle. Note that the ekpyrotic phase does not undo the dilution, since the scale factor shrinks very modestly during ekpyrosis.

It is useful to keep track of various scales during the evolution of a given cycle. We are now at the start of a dark energy phase, during which the scale factor a grows by an amount $e^{\mathcal{N}_{DE}}$, where, by definition, \mathcal{N}_{DE} is the number of e-folds of dark energy domination. During this phase, the Hubble parameter remains approximately constant. Then, during the ekpyrotic phase, the scale factor shrinks very modestly, by $|V_{end}/V_{beg}|^{-p/2} \sim \mathcal{O}(1)$, where V_{beg} is the value of the potential at the beginning of the ekpyrotic phase; this value coincides with the value of the present day cosmological constant $V_{beg} \approx V_0$. During the same time, the Hubble parameter grows immensely, by an amount $|V_{end}/V_{beg}|^{1/2} \equiv e^{\mathcal{N}_{ek}}$, where \mathcal{N}_{ek} denotes the number of e-folds of ekpyrosis. As the branes collide, radiation and matter is produced and the branes “reheat” with a finite temperature, where we define T_r to be the temperature at the time that the radiation density equals the scalar kinetic energy density. There is a contracting kinetic energy phase just before the brane collision, followed by an expanding kinetic phase after the bang. These two phases are almost

exactly symmetrical, except for a slight boost that the scalar field obtains at the bounce. This boost must occur in order to achieve a cyclic model, because, due to extra Hubble damping from the radiation, the scalar needs a slight excess of kinetic energy to cross the potential well onto the plateau, see figure 10. There are two known ways by which the scalar field can obtain this slight excess energy [93,104]: either due to the coupling of the matter fields to the radion, or if there is slightly more matter produced on the negative-tension brane than on the positive-tension one. The latter result makes sense heuristically, as more negative than positive inertia is produced in that case. The first expanding kinetic phase is followed by a brief, and unimportant [35], $w \gg 1$ phase, which in turn is followed by another expanding kinetic phase. We can treat all of these phases as a single kinetic phase, as in [35]. Then, since $\rho \propto a^{-6} \propto t^{-2}$ during kinetic energy domination, the net expansion of the scale factor is given by $(|V_{end}|^{1/4}/T_r)^{2/3} = e^{2\gamma_{KE}/3}$, where we define the parameter

$$\gamma_{KE} \equiv \ln \left(\frac{|V_{end}|^{1/4}}{T_r} \right). \quad (193)$$

Meanwhile, the Hubble parameter has shrunk by $(|V_{end}|^{1/4}/T_r)^{-2} = e^{-2\gamma_{KE}}$. Finally, during radiation and matter domination, the universe grows by a factor of $T_r/T_0 \equiv e^{N_{rad}}$, where T_0 is the temperature of the CMB today; at the same time, the Hubble parameter shrinks by $(T_r/T_0)^{-2} = e^{-2N_{rad}}$. Therefore, to recapitulate, the scale factor a grows by a total of

$$\mathcal{N}_{DE} + \frac{2\gamma_{KE}}{3} + N_{rad} \quad (194)$$

e-folds over the course of a single cycle. However, because

$$\frac{(-V_{end})^{1/2}}{V_{beg}^{1/2}} \frac{T_r^2}{(-V_{end})^{1/2}} \frac{T_0^2}{T_r^2} \approx 1, \quad (195)$$

due to $V_{beg} \approx T_0^4$, the Hubble parameter actually returns to its original value after one cycle. This result can also be re-written as the statement that [35]

$$\mathcal{N}_{ek} \approx 2(\gamma_{KE} + N_{rad}). \quad (196)$$

There is no constraint on the number of e-folds of dark energy domination \mathcal{N}_{DE} .

In order for the cyclic model to work, the temperature T_r has to satisfy a number of constraints. The most obvious one is that the reheat temperature has to be high enough for nucleosynthesis to take place. Also, it should be below the GUT scale, so that no topological defects are formed. Hence the first constraint is

$$1\text{MeV} < T_r < 10^{16}\text{GeV}. \quad (197)$$

Next, in order for the successful predictions of nucleosynthesis not to be adversely affected by the overproduction of gravitational waves [5,90], the temperature has to satisfy [17]

$$T_r \geq \frac{p}{20} |V_{end}|, \quad (198)$$

where p is defined in (20) and is typically of $\mathcal{O}(10^{-3})$ for realistic models. Also, the radiation component should not dominate the cosmological evolution until the scalar field has crossed the potential well and gotten onto the plateau of the potential. Now, combining the evolution of the scalar field, equation (32), with the form of the ekpyrotic potential $V \sim -e^{-c\phi}$, we have that $V \propto t^{-\sqrt{2/3}c}$. Hence, the time t_{cross} it takes for the scalar field to cross the well (*i.e.* to go from $-V_{end}$ to V_0) is $t_{cross} = t_{end} |V_{end}/V_0|^{\sqrt{3/2}/c}$. We need the time of radiation domination t_r to be larger than t_{cross} , or, using that $t_r \approx H_r^{-1} \approx T_r^{-2}$, we get the constraint (this bound was derived in [51], although there is a typo regarding the sign of the inequality in the paper)

$$T_r \leq (-V_{end})^{1/4} \left| \frac{V_0}{V_{end}} \right|^{\sqrt{3p/16}}. \quad (199)$$

It should be noted that a broad range of parameters satisfies all the constraints for cycling [51]. Typical values for the various parameters are for example

$$|V_{end}| = (10^{15} GeV)^4, \quad T_r = 10^{12} GeV, \quad p = 10^{-3}. \quad (200)$$

Then, with $T_0 = 10^{-3} eV$, we have $\gamma_{KE} \approx 7$ and $N_{rad} \approx 55$. Hence this leads to more than 120 e-folds of ekpyrosis, largely sufficient to solve the standard cosmological puzzles. And assuming zero e-folds of dark energy domination, according to (194), the universe grows by a factor of about $e^{60} \approx 10^{26}$ per cycle. Note that this means that our currently observable universe measured only about a cubic meter one cycle ago!

The enormous time scales involved in the cyclic model (a single cycle can already easily last a trillion years) mean that new approaches are possible for addressing some long-standing problems, such as for example the cosmological constant problem. In trying to explain the smallness of the cosmological constant by a dynamical relaxation mechanism, the problem has typically been that there simply isn't enough time between the big bang and now in order to achieve the required level of tuning. This has been an issue with the model of Abbott [2] for example. However, in the cyclic model, the same model can work, as shown in [97]. The idea is that the plateau in the cyclic potential is really part of a tilted washboard-shaped potential for an axion-type scalar. That scalar can then tunnel down the potential, spending more and more time at lower and lower values of the cosmological constant. It spends an exponentially longer time at the lowest positive local minimum of the potential (before tunneling to negative values, where the universe then collapses), and hence that value of the cosmological constant is the most probable one. This

is just one example, but the general idea is that there could be evolutionary time scales vastly larger than the ones that are usually considered, and which can lead to qualitatively new explanations.

Let us turn to the question of “initial” conditions. The single-field cyclic model is an attractor, while at the same time managing to generate scalar perturbations that are nearly scale-invariant [93,42,39]. This is really quite remarkable, and it means that the issue of initial conditions is essentially circumvented. Evidently, avoiding the issue of initial conditions is one of the main attractions of cyclic models. However, for two-field models [61], the situation is more complicated. As discussed in section 4.1.2, in order to achieve a nearly scale-invariant spectrum of scalar perturbations, the background trajectory must fall off along a ridge of the potential, *i.e.* in the presence of an unstable transverse direction. This instability is essential to obtain perturbations that are compatible with observations [100], and hence there is an issue of initial conditions. This is an open problem at present. The question of how the field returns to a starting position very close to the ridge after each cycle remains to be addressed. Of course, to some extent this problem is linked to the general problem of moduli stabilization (or quasi-stabilization in the cosmological context). In heterotic string theory, this problem is less well understood than for example in the type IIB theory at present. More work is certainly needed in this area. In the more phenomenological context of the new ekpyrotic models, Buchbinder *et al.* [23] have proposed to include a stabilizing positive mass potential at the top of the ekpyrotic ridge, while coupling the entropy field to light fermions. Then, during this pre-ekpyrotic phase, the entropy field oscillates around the minimum of the potential. The field gets localized near the top of the ridge, because the oscillations are damped as energy is dissipated in the form of thermal radiation due to the coupling of the entropy field to light fermions. Something similar could also work in the cyclic model, and in fact, as the entropy field rolls slightly up the stabilizing potential, this increased potential energy could be identified with today’s cosmological constant⁶. Of course, one would like to see such a potential arise from a fundamental theory, and avoid putting it in by hand.

Finally, we will address the global structure of the cyclic universe and some more speculative questions. Since the universe undergoes an exponentially large net expansion every cycle, over large patches the universe will approximately have the structure of de Sitter space, but with singular hypersurfaces corresponding to the big crunch/big bang transitions [96]. Hence, starting from a given patch of spacetime, the Penrose diagram will look approximately like the one shown in Fig. 11. The cycling can however not be synchronous over super-horizon scales, as the ekpyrotic phase only homogenizes the branes over large, but finite-sized patches. Occasionally, there will be large quantum

⁶ This idea was suggested to the author by Paul Steinhardt.

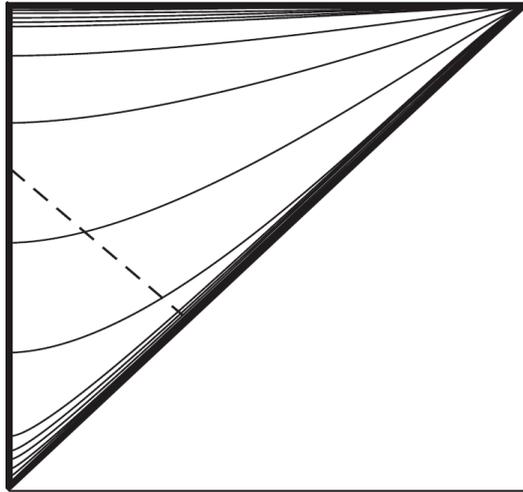


Fig. 11. On average, the Hubble parameter and the energy density are positive in the cyclic universe, and hence it can be represented approximately by the conformal diagram of de Sitter space, but with singular hypersurfaces corresponding to the big crunch/big bang transitions (here these are drawn only in the region covered by the usual flat coordinates on de Sitter space). In this diagram, time is vertical, and space horizontal, with each point representing a 2-sphere. A representative light signal is represented by the dashed line - it travels through many crunches, which makes it unlikely that one could see back through many cycles, even in principle.

fluctuations as the brane collision is approached, leading to the formation of a large black hole at the collision. In this region, the cycling will stop [35]; but since there is such a huge expansion of the universe during each cycle, these regions will be exponentially rare. In the same vein, one might ask whether an advanced civilization, with sufficient control over the spacetime metric, could stop the cycling in order to prolong its lifetime. Presumably so, but due to causality, only the region in its vicinity would be affected, and again, due to the vast net expansion, there would be plenty of room left for the cycling to continue unaffected. Thus, during each cycle, vast new regions of space are created, but within the same universe; in contrast to eternal inflation [70], there is no proliferation of universes. Note also that it seems likely that only information from the previous cycle is remembered, and not from the ones before [35]. This is essentially due to the effectiveness of the ekpyrotic phase in washing out all classical inhomogeneities, while new inhomogeneities are produced by random quantum processes. Nevertheless, could an advanced civilization have left us a message from the previous cycle? This seems extremely unlikely, as it would need to have had access to precisely the one cubic meter of space that has now become our currently observable universe. Could there have been a beginning to the cycling, for example if the two branes tunneled into existence? This is certainly possible, but if one really cannot see back more

than one cycle prior to the current one, then this question might be rather meaningless. Finally, we note that, according to the present understanding, there seems to be no limit to the number of cycles allowed⁷. Hence the cycles can continue indefinitely, creating more and more space, while injecting new matter and life into the universe periodically.

6 The Link to Fundamental Theory: The Embedding in Heterotic M-theory

It is the link between 11-dimensional supergravity and heterotic string theory which led to the colliding branes picture in the first place, and even though many of the relevant calculations are done in a 4-dimensional effective field theory, the higher-dimensional picture is of crucial importance for a deeper understanding of ekpyrotic/cyclic cosmology. Moreover, it is helpful in trying to visualize many of the processes described in the rest of this review, as it provides a geometrical interpretation of these⁸.

The starting point for both ekpyrotic and cyclic cosmology is the brane picture of our universe as suggested by string theory. The various string theories and 11-dimensional supergravity are related to each other by a web of dualities [108], and so any particular one of these theories could be used as a model for the early universe. However, certain theories have proved more intuitive than others in tackling particular problems, and it is the duality between 10-dimensional heterotic string theory with gauge group $E_8 \times E_8$ and 11-dimensional supergravity [43,44] that has inspired the early universe models described in this review. Neglecting fermions, the action of the maximal, 11-dimensional supergravity is given by [31]

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int_{M^{11}} \sqrt{-\hat{g}} \left[\hat{R} - \frac{1}{24} \hat{G}_{MNPQ} \hat{G}^{MNPQ} - \frac{\sqrt{2}}{1728} \epsilon^{M_1 \dots M_{11}} \hat{A}_{M_1 M_2 M_3} \hat{G}_{M_4 \dots M_7} \hat{G}_{M_8 \dots M_{11}} \right], \quad (201)$$

⁷ Theorems about past-incompleteness [15] as well as entropy bounds [37] do not apply, because of the singular crunch/bang transitions [96].

⁸ Many early studies dealing with the embedding of ekpyrotic/cyclic models in string theory/supergravity have considered the space in between the branes to be a slice of Anti de Sitter spacetime, see *e.g.* [48,93,99,78]. Obtaining 5-dimensional AdS in supergravity is straightforward [88], but as soon as the branes are added, the story becomes much more complicated, and not all issues have been resolved at present [25,33,32,63]. This is why we only present the theoretically better-motivated embedding into heterotic M-theory here.

where we denote 11-dimensional quantities with hats. κ_{11} determines the strength of gravity in 11 dimensions. Here $\hat{G}_{MNPQ} = 4!\partial_{[M}\hat{A}_{NPQ]}$ is the 4-form field strength associated with the 3-form \hat{A}_{MNP} . The $\hat{A}_{(3)}\hat{G}_{(4)}\hat{G}_{(4)}$ term is a Chern-Simons term and it is topological in nature. It will be unimportant in what follows. 11-dimensional supergravity is a classical theory, which nobody knows how to quantize. However, we can calculate anomalies (which do not depend on the microscopic structure of spacetime), and these allow us a glimpse of quantum M-theory. If we consider the 11-dimensional spacetime to take the form of a 10-dimensional spacetime plus a line segment, then anomaly cancelation requires us to add the following boundary terms [43,44]

$$S_{boundary} = -\frac{1}{8\pi\kappa_{11}^2} \left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} \int_{M_{10}^{(1)}} \sqrt{-g} \left[\text{tr}(F^{(1)})^2 - \frac{1}{2}\text{tr}R^2 \right] - \frac{1}{8\pi\kappa_{11}^2} \left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} \int_{M_{10}^{(2)}} \sqrt{-g} \left[\text{tr}(F^{(2)})^2 - \frac{1}{2}\text{tr}R^2 \right]. \quad (202)$$

The gauge fields correspond to the group E_8 and the $\text{tr}R^2$ terms are required by supersymmetry. In the limit that the line segment becomes vanishingly small, the two boundaries sit on top of one another, and we recover the supergravity approximation to the 10-dimensional $E_8 \times E_8$ heterotic string theory. In fact, the size of the line segment is related to the coupling constant of the heterotic theory (roughly because the size of the line segment is reinterpreted as the dilaton field in 10 dimensions), via the famous relation [108]:

$$R_{11} = g_s^{\frac{2}{3}} \quad (203)$$

where R_{11} is the radius of the 11th dimension and g_s is the string coupling constant. Thus, at strong coupling, the world looks 11-dimensional. As the boundary branes collide, the 11th dimension disappears, but at the same time the coupling goes to zero. Hence one might expect the collision to be rather mild.

In order to get a realistic model of particle physics, we can now consider curling up 6 of the spatial dimensions into a Calabi-Yau manifold. Then it turns out that in order to obtain the right magnitude of Newton's constant in 4 dimensions, the line segment must be larger than the radius of the Calabi-Yau manifold by a factor of about 30 or so [8,109]. Hence, going up in energy, the world looks 4-dimensional, then 5-dimensional (as the line segment becomes visible), and only above the GUT scale 11-dimensional. This is the reason for choosing a 5-dimensional braneworld setup in describing the dynamics of the universe close to the big bang. The relevant 5-dimensional theory is called heterotic M-theory, and it can be obtained by dimensionally reducing the 11-dimensional theory described above on a 6-dimensional Calabi-Yau manifold,

as follows [72,73]. The metric is reduced according to

$$ds_{11}^2 = V_{CY}^{-2/3} g_{mn} dx^m dx^n + V_{CY}^{1/3} g_{ab} dx^a dx^b . \quad (204)$$

where V_{CY} denotes the volume of the Calabi-Yau, g_{ab} being its metric. Indices m, n, \dots run over 5 spacetime dimensions (including the line segment direction, labeled 11, but where we also use the coordinate $x^{11} = y$), and a, b, \dots denote Calabi-Yau directions. Due to the boundary terms (202), there is a modified Bianchi identity for the 4-form mode, which has to satisfy

$$(d\hat{G})_{11MNPQ} = -\frac{1}{2\sqrt{2}\pi} \left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} \left\{ J^{(1)}\delta(y-1) + J^{(2)}\delta(y+1) \right\}_{MNPQ} \quad (205)$$

where the currents are given by

$$J^{(i)} = \text{tr} F^{(i)} \wedge F^{(i)} - \frac{1}{2} \text{tr} R \wedge R \quad (206)$$

and where we have chosen the coordinate positions $y = \pm 1$ for the location of the boundary branes. In the weakly coupled heterotic string theory the Bianchi identity reads

$$dG \sim \text{tr} R \wedge R - \text{tr} F^{(1)} \wedge F^{(1)} - \text{tr} F^{(2)} \wedge F^{(2)} . \quad (207)$$

The so-called ‘‘standard embedding’’ of the gauge connection into the spin connection ensures that

$$\text{tr} F^{(1)} \wedge F^{(1)} = \text{tr} R \wedge R \quad (208)$$

and together with setting $F^{(2)} = 0$ we see that we can have $dG = 0$ and thus set G to zero consistently. In our case, there appears to be no way of achieving $\text{tr} F^{(i)} \wedge F^{(i)} = \frac{1}{2} \text{tr} R \wedge R$, and thus there will always be some components of \hat{G} that are required to be non-zero. This mode is called the non-zero mode of the reduction. The standard embedding (208) then leads to

$$(d\hat{G})_{11MNPQ} = -\frac{1}{4\sqrt{2}\pi} \left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} (\delta(y-1) - \delta(y+1)) \{ \text{tr} R \wedge R \}_{MNPQ} . \quad (209)$$

The interpretation of this equation is that the $\pm \text{tr} R \wedge R$ terms are sources for oppositely charged branes sitting at the endpoints ($y = \pm 1$) of the line segment. We can solve the modified Bianchi identity (209) above (and at the same time satisfy the equation of motion for \hat{G}) to lowest order by setting⁹

$$\hat{G}_{abcd} = \frac{\alpha}{\sqrt{2}} \epsilon_{abcd}{}^{ef} \omega_{ef} \theta(y) , \quad (210)$$

⁹ Compared to [72,73], we have re-scaled α such that $\alpha = \alpha_{\text{LOSW}}/3\sqrt{2}$. Note that the parameters α and β used in this section are unrelated to the use of the same letters as indices of Hankel functions in section 4.

where

$$\theta(y) = \begin{cases} +1 & \text{for } -1 \leq y < +1 \pmod{4} \\ -1 & \text{for } -3 < y < -1 \pmod{4} \end{cases} \quad (211)$$

and

$$\alpha = \frac{\pi}{6} \left(\frac{\kappa_{11}}{4\pi} \right)^{\frac{2}{3}} \beta \quad (212)$$

where $\beta = -\frac{1}{8\pi^2} \int_{\mathcal{C}} \text{tr} R \wedge R$ is the first Pontryagin class of the Calabi-Yau (the integration being done over the (2,2)-cycle \mathcal{C} corresponding to the Kähler form ω_{ab}). Note that \hat{G}_{abcd} is related to the integer β and thus \hat{G} is quantized; hence α is also quantized. We are only considering the lowest mode in the (Fourier) expansion of the full solution to the modified Bianchi identity, as all higher order modes are massive (their mass being of the order of the Calabi-Yau volume, *i.e.* the GUT scale) and decouple in the approximation considered here. In contrast to the other modes arising in the dimensional reduction of \hat{G} , the non-zero mode cannot be set to zero.

Keeping only gravity, the non-zero mode and the Calabi-Yau volume modulus in 5 dimensions (which is a consistent truncation), we end up with the action

$$S_5 = \frac{1}{2\kappa_5^2} \int_{5d} \sqrt{-g} \left[R - \frac{1}{2} V_{CY}^{-2} \partial_m V_{CY} \partial^m V_{CY} - 6\alpha^2 V_{CY}^{-2} \right] \\ + \frac{1}{2\kappa_5^2} \left\{ -12\alpha \int_{4d, y=+1} \sqrt{-g} V_{CY}^{-1} + 12\alpha \int_{4d, y=-1} \sqrt{-g} V_{CY}^{-1} \right\}. \quad (213)$$

One should especially note the presence of the potential term $6\alpha^2/V_{CY}^2$ which arose from the non-zero mode. This potential means that (1,4)-dimensional Minkowski space is not a solution to our theory, and hence not the vacuum. However such a ‘‘cosmological’’ term is dual in 5 dimensions to a 5-form field strength, which can support a 3-brane solution. In 5 dimensions a 3-brane is of codimension 1 and thus represents a domain wall. Note that in general for a 3-brane solution to be completely consistent, one needs a (4-dimensional) source action. But we actually have two 4-dimensional sources in the action (213) located at $y = \pm 1$. This double domain wall vacuum was found in [72]. It is given by

$$ds^2 = h^{2/5}(y) \left[A^2 (-d\tau^2 + d\vec{x}^2) + B^2 dy^2 \right], \\ V_{CY} = B h^{6/5}(y), \\ h(y) = 5\alpha y + C, \quad (214)$$

where A , B and C are arbitrary constants. The y coordinate is taken to span the orbifold (line segment) S^1/\mathbb{Z}_2 with fixed points at $y = \pm 1$. In an ‘extended’

picture of the solution, obtained by \mathbb{Z}_2 -reflecting the solution across the branes, there is a downward-pointing kink at $y = -1$ and an upward-pointing kink at $y = +1$. These ensure the Israel conditions are satisfied, with the negative-tension brane being located at $y = -1$ and the positive-tension brane at $y = +1$. In the static domain wall solution above, the volume of the Calabi-Yau manifold and the distance between the boundary branes are determined in terms of the moduli B and C , while the scale factors on the branes are determined in terms of A and C . The modulus C additionally determines the height of the harmonic function h at a given position in y . Before continuing by describing how we can extend this solution to allow for motion of the branes, we should add some remarks: by considering only the boundary branes, we are looking at a universal, but simplified version of heterotic M-theory. Typically, there are many more fields that can play a role, and there can be additional branes in the bulk. Such extra ingredients are usually necessary in order to obtain a realistic model of particle physics on the branes, see *e.g.* [16,21]. Here we focus on the simplest case of having only boundary branes, since we are interested in the collision of these boundary branes. Also, the braneworld picture suggest a natural place for dark matter to reside, namely on the second boundary brane. Any form of matter residing on the “other” brane will appear as dark to us, since we would only detect its effects via gravity. However, there is no quantitative argument known for how much of the dark matter should be ordinary matter on second brane, or weakly-interacting matter on the visible brane.

We are interested in finding the effective 4-dimensional theory describing the motion of the boundary branes that we have just discussed. We will obtain this by the method of the moduli space approximation [60]: we promote the moduli A, B, C in the solution above to arbitrary functions of the brane conformal time τ , yielding the ansatz:

$$\begin{aligned} ds^2 &= h^{2/5}(\tau, y) \left[A^2(\tau) (-d\tau^2 + d\vec{x}^2) + B^2(\tau) dy^2 \right], \\ V_{CY} &= B(\tau) h^{6/5}(\tau, y), \\ h(\tau, y) &= 5\alpha y + C(\tau), \quad -1 \leq y \leq +1. \end{aligned} \tag{215}$$

Let us give a brief justification for this ansatz: firstly, we note that the ansatz satisfies the τy Einstein equation identically. This is important, since otherwise the τy equation would act as a constraint, see *e.g.* [40]. Secondly, there is no $g_{\tau y}$ modulus, since this metric component is odd under the \mathbb{Z}_2 symmetry, and therefore has to vanish at the location of the branes. Any such component which is zero at the location of the branes, but non-zero in the bulk, is necessarily massive. In fact, from the work of [62], we know that, apart from the above moduli, all other perturbations have a positive mass squared. Having defined the time-dependent moduli, we would now like to derive the action summarizing their equations of motion. This is achieved by simply plugging

the ansatz (215) into the action (213), yielding the result

$$S_{\text{mod}} = -6 \int_{4d} A^2 B I_{\frac{3}{5}} \left[\left(\frac{A'}{A} \right)^2 - \frac{1}{12} \left(\frac{B'}{B} \right)^2 + \frac{A'B'}{AB} - \frac{1}{25} \frac{I_{-\frac{7}{5}}}{I_{\frac{3}{5}}} C'^2 + \frac{3}{5} \frac{I_{-\frac{2}{5}} A' C'}{I_{\frac{3}{5}} A} \right], \quad (216)$$

where we have defined

$$I_n = \int_{-1}^1 dy h^n = \frac{1}{5\alpha(n+1)} [(C+5\alpha)^{(n+1)} - (C-5\alpha)^{(n+1)}]. \quad (217)$$

This action can be greatly simplified by introducing the field redefinitions

$$a^2 \equiv A^2 B I_{\frac{3}{5}}, \quad (218)$$

$$e^{\phi_1/\sqrt{2}} \equiv B (I_{\frac{3}{5}})^{3/4}, \quad (219)$$

$$\phi_2 \equiv -\frac{\sqrt{6}}{20} \int dC (I_{\frac{3}{5}})^{-1} \left[9 (I_{-\frac{2}{5}})^2 + 16 I_{-\frac{7}{5}} I_{\frac{3}{5}} \right]^{1/2}. \quad (220)$$

Note that a has the interpretation of being roughly the four-dimensional scale factor, whereas ϕ_1 and ϕ_2 are four-dimensional scalars. The definition (220) can be rewritten as stating that

$$d\phi_2 = -\frac{\sqrt{6}dC}{2(C+5\alpha)^{1/5}(C-5\alpha)^{1/5}I_{\frac{3}{5}}}. \quad (221)$$

This expression can be integrated to yield

$$C = 5\alpha \left[\frac{(1 + e^{2\sqrt{2/3}\phi_2})^{5/4} + (1 - e^{2\sqrt{2/3}\phi_2})^{5/4}}{(1 + e^{2\sqrt{2/3}\phi_2})^{5/4} - (1 - e^{2\sqrt{2/3}\phi_2})^{5/4}} \right]. \quad (222)$$

In terms of a , ϕ_1 and ϕ_2 , the moduli space action (216) then reduces to the remarkably simple form [60]

$$S_{\text{mod}} = \int_{4d} [-6a'^2 + a^2(\phi_1'^2 + \phi_2'^2)]. \quad (223)$$

The minus sign in front of the kinetic term for a is characteristic of gravity, and in fact this is the action for gravity with scale factor a and two minimally coupled scalar fields that is used for the calculations of cosmological perturbations in section 4. There is also a manifest $O(2)$ rotation symmetry for the scalar fields. The equation of motion for a reads

$$6a'' = -a(\phi_1'^2 + \phi_2'^2), \quad (224)$$

while the equations of motion for ϕ_1 and ϕ_2 immediately lead to the conserved charges Q_1 and Q_2 , according to

$$a^2 \phi_1' = Q_1, \quad a^2 \phi_2' = Q_2. \quad (225)$$

The solutions to these equations are given by

$$a^2 = 2 \sqrt{Q_1^2 + Q_2^2} (\tau - \tau_a), \quad (226)$$

$$\psi = \frac{\sqrt{\frac{3}{2}} Q_1}{\sqrt{Q_1^2 + Q_2^2}} \ln[\phi_{1,0}(\tau - \tau_a)], \quad (227)$$

$$\chi = \frac{\sqrt{\frac{3}{2}} Q_2}{\sqrt{Q_1^2 + Q_2^2}} \ln[\phi_{2,0}(\tau - \tau_a)], \quad (228)$$

where Q_1 , Q_2 , τ_a , $\phi_{1,0}$, and $\phi_{2,0}$ are constants of integration.

We can now return to the ansatz (215) and relate physical quantities in five dimensions to the moduli fields a , ϕ_1 and ϕ_2 : if we denote the distance between the branes by d , and the volume of the Calabi-Yau and the brane scale factors at the locations $y = \pm 1$ by $V_{CY\pm}$ and a_{\pm} respectively, then we have the relations

$$d = \frac{1}{3(2\alpha)^{1/4}} e^{\phi_1/\sqrt{2} - \sqrt{3/2}\phi_2} [(1 + e^{2\sqrt{2/3}\phi_2})^{3/2} - |1 - e^{2\sqrt{2/3}\phi_2}|^{3/2}] \quad (229)$$

$$V_{CY\pm} = (2\alpha)^{3/4} e^{\phi_1/\sqrt{2}} \begin{cases} (\cosh \sqrt{2/3}\phi_2)^{3/2} \\ (-\sinh \sqrt{2/3}\phi_2)^{3/2} \end{cases}, \quad (230)$$

$$a_{\pm} = (2\alpha)^{1/8} a e^{-\phi_1/2\sqrt{2}} \begin{cases} (\cosh \sqrt{2/3}\phi_2)^{1/4} \\ (-\sinh \sqrt{2/3}\phi_2)^{1/4} \end{cases}. \quad (231)$$

These relations are useful in interpreting particular solutions to the moduli equations of motion. Note in particular that the 4-dimensional effective theory describes the motion of *both* branes at the same time, as well as the geometry of the 5-dimensional bulk. In the limit that $\phi_2 \rightarrow +\infty$, we have

$$d \simeq (2\alpha)^{-1/4} e^{\phi_1/\sqrt{2} - \phi_2/\sqrt{6}}, \quad (232)$$

$$V_{CY\pm} \simeq (2\alpha)^{3/4} e^{\phi_1/\sqrt{2} + \sqrt{3/2}\phi_2}, \quad (233)$$

whereas for $\phi_2 \rightarrow -\infty$, we have

$$d \simeq (2\alpha)^{-1/4} e^{\phi_1/\sqrt{2} + \phi_2/\sqrt{6}}, \quad (234)$$

$$V_{CY\pm} \simeq (2\alpha)^{3/4} e^{\phi_1/\sqrt{2} - \sqrt{3/2}\phi_2}. \quad (235)$$

Thus, in both limits, $\ln d$ and $\ln V_{CY\pm}$ are orthogonal variables. This means that, sufficiently far away from the $\phi_2 = 0$ axis, the fields ϕ_1 and ϕ_2 are, up to a re-scaling, simply related to $\ln d$ and $\ln V_{CY\pm}$ by a rotation in field space. Since the moduli space trajectories in terms of ϕ_1 and ϕ_2 are straight lines, far from the $\phi_2 = 0$ axis, the trajectories will also be approximately straight lines in terms of $\ln d$ and $\ln V_{CY\pm}$.

The $\phi_2 = 0$ axis plays a special role, in that it represents a boundary to moduli space. Positive values of ϕ_2 would lead to an unphysical negative volume of the Calabi-Yau manifold at the location of the negative-tension brane, and are forbidden. Physically, the boundary corresponds to a naked singularity in the bulk spacetime [67]. In fact, it can be shown that in the presence of a wide variety of matter fields on the negative-tension brane, the negative-tension brane bounces off this singularity in a smooth way and without encountering the singularity [67]. This is only possible because of the special properties of gravity on an object of negative tension. In comparing the moduli to higher-dimensional quantities via equations (229)-(231), it is apparent that this bounce of the negative-tension brane is equivalent to flipping the sign of ϕ_2 and thus ϕ'_2 also, and hence the otherwise straight trajectories in scalar field space get reflected off the $\phi_2 = 0$ boundary, as shown in Fig. 6. This reflection has the important consequence of converting entropy perturbations around the background evolution into curvature perturbation, as discussed in section 4.1.2.

As we have just seen, the general solution to the 4-dimensional effective theory (223) is a straight line of arbitrary slope in scalar field space, but bending at the boundary $\phi_2 = 0$. In general, from the higher-dimensional point of view, the solutions correspond to a Kasner spacetime near the brane collision. However, one particular slope is special in that it corresponds to the least singular colliding branes solution studied in [59]. It is given by

$$a = |2y_0\tau|^{1/2}, \quad e^{\phi_1} = |2y_0\tau|^{3/2\sqrt{2}}, \quad e^{\phi_2} = |4\alpha y_0\tau|^{\sqrt{3}/2\sqrt{2}}, \quad (236)$$

where $2y_0$ has the interpretation of being the velocity of the branes at the collision, as, for small τ , we have from (229) that $d \approx 2y_0\tau$. For this solution, the brane scale factors and the Calabi-Yau volume are finite at the brane collision (as can be verified directly using (229)-(231)), and hence the spacetime metric asymptotes to Milne near the collision. It was argued in section 3 that we can take this solution as a model spacetime for the crunch/bang transition. As discussed there, the ekpyrotic phase ensures that the curvatures remain small until the quantum gravity regime is reached. In particular, higher-derivative corrections involving the Riemann tensor will also be small near the collision

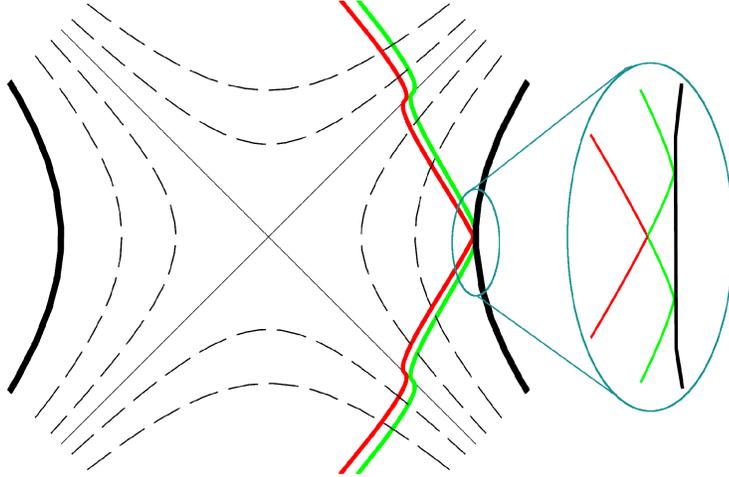


Fig. 12. A Kruskal-type spacetime diagram in which the exact trajectories of the positive- and negative-tension branes are plotted. The negative-tension brane (in green) is always to the right of the positive-tension brane (in red), thereby shielding it from the naked singularity (thick black line). The bounces of the negative-tension brane off the naked singularity, as well as the collision of the branes themselves, are shown at a magnified scale in the inset. Note that the physical spacetime is restricted to the region bounded by the two branes.

for the quasi-Milne spacetime, and hence we should be able to trust the above solution up to about a Planck time (or so) on either side of the collision.

Note that the solution above is simply a solution to the moduli space effective action, and hence is only the leading order solution in an expansion in the brane velocity. However, the corresponding exact solution may actually be found directly in 5-dimensional heterotic M-theory (213) [59]. Above, we have used a gauge in which the bulk is dynamical and the branes are kept at fixed coordinate positions. Here, it will be more useful to use a gauge where we have a static bulk through which the branes are moving. Then, subject to the boundary condition that the spacetime asymptotes to Milne at the collision, one can derive a Birkhoff-type theorem which fixes the bulk metric to be given by [59]

$$\begin{aligned}
 ds^2 &= -f dT^2 + \frac{r^{12}}{f} dr^2 + r^2 d\vec{x}^2, \\
 f(r) &= \alpha^2 r^2 - \mu r^4, \quad V_{CY} = r^6,
 \end{aligned}
 \tag{237}$$

where $0 \leq r \leq \sqrt{\mu}/\alpha$, and the coordinate T is unrelated to the time coordinate appearing in the section 3.2. Physically, this solution describes a timelike naked singularity located at $r = 0$. The coordinates above do not cover the whole spacetime manifold, but the maximal extension may easily be constructed

following the usual Kruskal procedure [59]. The branes then move in this background spacetime according to the Israel junction conditions, and this motion is shown in Fig. 12 (for a detailed derivation see [59]). One finds that the induced brane metrics are indeed cosmological, with scale factors given by

$$a_{\pm} = (1 \pm 4\sqrt{\mu}\tau_{\pm})^{1/4}, \quad (238)$$

where τ_{\pm} is the brane conformal time, and we have re-scaled a_{\pm} to unity at the collision, which is taken to occur at $\tau_{\pm} = 0$. Upon setting $\mu = 0$, we immediately recover the static domain wall solution (214), after a suitable change of coordinates. More generally, we require $\mu \geq 0$ to avoid the appearance of imaginary scale factors. μ is directly related to the brane velocity y_0 , and one can verify that the scaling solution (236) is recovered as an approximation at low velocity [60]. For $\mu > 0$, starting from the brane collision, the two branes proceed to separate. However, while the positive-tension brane travels out to large radii unchecked, the negative-tension brane reaches the naked singularity (at which a_- and V_{CY-} tend to zero) in a finite brane conformal time $\tau_- = (4\sqrt{\mu})^{-1}$. The resulting singularity is simple to regularize. As stated above, if almost any type of well-behaved matter is present on the negative-tension brane – even in only vanishing quantities – then, rather than hitting the singularity, the negative-tension brane will instead undergo a bounce at some small finite value of the scale factor and move away from the singularity. We can make this a little bit more precise by looking at the Friedmann equation on the negative-tension brane. For the case in which a time-dependent scalar field (with a V_{CY} coupling to the Calabi-Yau volume scalar) is present on the brane, this equation takes the form (see [59])

$$H_-^2 = \frac{\dot{a}_-^2}{a_-^2} = -2\alpha \frac{\rho_-}{a_-^{18}} + \frac{\rho_-^2}{a_-^{24}} + \frac{\mu}{a_-^{10}}, \quad (239)$$

where the constant ρ_- parameterizes the scalar kinetic energy density. The key feature of this equation is the negative sign in front of the first term on the right-hand side, reflecting the fact that matter on the negative-tension brane couples to gravity with the wrong sign. For sufficiently large values of the scale factor the right-hand side is dominated by the μa_-^{-10} term. If we further assume that the matter density on the branes is small compared to the brane tension (as is in any case necessary for the existence of a four-dimensional effective description, *i.e.* $\rho_- \ll \alpha$), so that the term linear in ρ_- dominates over the quadratic term, then it follows that at some small value of the scale factor the entire right-hand side must vanish. Thus a negative-tension brane, initially traveling towards the singularity, will generically undergo a smooth bounce at some small value of the scale factor. After this bounce the brane travels away from the singularity back towards large values of the scale factor. This behavior is specific to the negative-tension brane, and moreover, persists even in the limit in which ρ_- (and hence the initial matter density) is negligibly small. It can also be generalized to a wide class of negative-tension branes and

matter types [67].

In order for the cyclic model and the ekpyrotic mechanism to work, we must assume a force between the boundary branes expressed by a potential of the form shown in Fig. 10. Could a potential of this shape arise from the heterotic M-theory setting under discussion here? First of all, we should note that a negative potential is completely natural in supergravity (hence all the problems in trying to explain the observed dark energy). Further, one can imagine that an attractive force between the branes could arise from virtual exchange of membranes stretching between the boundary branes (see [66] for an example of such membrane solutions). Preliminary studies do indicate that an attractive force can arise from open membrane instantons [80,68] in a certain parameter range. Unfortunately, a complete calculation of the potential involving all non-perturbative effects has been impossible to do so far. The vanishing of the potential near the brane collision seems natural, as the string coupling constant g_s goes to zero there. Hence one would expect non-perturbative effects to vanish also as $\sim e^{-1/g_s}$ [93]. Certainly, one would eventually like to see a complete derivation of the potential in heterotic M-theory or a related setup.

7 Conclusions and Outlook

Postulating a slowly contracting phase of the universe with equation of state $w \gg 1$ (preceding the standard expanding phase of big bang cosmology) has extremely powerful consequences: it renders the universe flat and homogeneous over large patches, it delays the onset of chaos until the quantum gravity regime is reached, it prepares the universe ideally for a big crunch/big bang transition, and, on top of that, it generates small-scale scalar perturbations with a nearly scale-invariant spectrum. Hence, even if the higher-dimensional picture and the link to string theory seem unappealing to you, at the very least the ekpyrotic phase represents a new mechanism for solving the standard cosmological puzzles and a new way of generating cosmological perturbations. Moreover, these perturbations contain a significant non-gaussian component, so that they can be tested observationally and compared to simple, gaussian, inflationary models. Further into the future, the prediction of a blue small-amplitude gravitational spectrum will also be tested against observations.

But more generally, ekpyrotic and cyclic models present a very broad conceptual framework, in which the big bang is not the beginning of time, but rather a physical event amenable to a testable physical description. Cyclic models allow for evolutionary timescales much larger than the time since the last big bang, and can provide qualitatively new explanations in this way. The embedding of ekpyrotic/cyclic models in string theory provides a geometric interpretation for many cosmological processes, and can eventually provide

observational tests of the theory. Many open issues remain, which should be tractable within string theory: in particular, the issue of moduli stabilization requires further work, as does the closely related question of “initial” conditions in the presence of many moduli. Also, the all-important cyclic potential has not yet been derived from first principles. Most promising is the recent realization that the brane collision, and the corresponding 4-dimensional big crunch/big bang transition, can perhaps be understood within the context of generalizations of the non-perturbative gauge/gravity correspondence. Much work remains to be done here before a physically realistic case can be treated; but this is a topic of such importance to cosmology that we should see this as an opportunity, rather than a drawback. It would be truly magnificent if we could find observational signatures of higher-dimensional physics, and of a time before the big bang.

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