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Impact of demand uncertainty on supply chain cooperation of single-period products

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Abstract

Cooperation is an approach of improving competitive advantages of a supply chain. A two-echelon supply chain consisting of a manufacturer and a retailer for a single-period product is studied, and retail-market demand uncertainty is described by coefficient of variation. We develop a cooperation mechanism to address the cooperation and its implementation between the manufacturer and the retailer, two market situations are considered: (i) the wholesale price and the order quantity are decision variables, (ii) the wholesale and the retail prices as well as the order quantity are decision variables. In both market situations, our research shows that: (1) the cooperation mechanism can improve the overall channel profits and the supply chain members' allocated profits, (2) the described cooperation is conditional on retail-market demand uncertainty: it can be implemented if, and only if, the fluctuation of retail-market demand is relatively small and coefficient of variation of retail-market demand does not exceed an upper bound. Impacts of retail-market demand uncertainty on wholesale price, order quantity and/or retail price have also been investigated through analytical and numerical analyses. Although our research is based on the assumption that the manufacturer dominates the supply chain in the non-cooperative situation, which is not the case for most retailer-driven supply chains, this research is still significant on providing guidelines for practitioners in current China mid-level car market that is similar to situations described in the paper.

Keywords: Supply chain; Newsvendor problem; Demand uncertainty; Cooperation

1. Introduction

Single-period products are the products that have only one chance to order and sell in single period. The classical single-period problem, also known as the newsboy or newsvendor problem, is to find a product's order quantity that maximizes the expected profit under probabilistic demand. The single-period problem

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assumes that if any inventory remains at the end of the period, a discount is used to sell it or it is disposed of (Khouja, 1999). If the order quantity is smaller than the realized demand, the newsvendor forgoes some profit and incurs a loss-of-goodwill (Dominey and Hill, 2004). The single-period problem is reflective of many real life situations and is often used to aid decision-making in the fashion and sporting industries, both at the manufacturing and retail levels (Gallego and Moon, 1993; Wang and Benaroch, 2004). Because of the high risk in production and business operation of this kind of products, it is important for the decision maker to realize the impact of retail-market demand uncertainty.

Demand uncertainty of single-period problems has been the subject of many recent researches (Mantrala and Raman, 1999). But most of these researches assume non-cooperative (i.e., leader–follower) relationship between manufacturer(s) and retail(s), in which manufacturer is the leader and retailer is the follower. Li et al. (1996), Chen and Xu (2001) studied conflict and coordination of a manufacturer and a retailer in a supply channel for single-period products, and provided ways of reducing negative impacts of demand uncertainty on the supply channel. Iyer and Bergen (1997) studied the effect of "demand uncertainty" in a manufacturer–retailer channel, they assumed that retail price and wholesale price are fixed, the manufacturer makes no decision, and retailer's order quantity is the only decision variable—as in the classical newsboy problem. Lau and Lau (2002) considered a non-cooperative game between a manufacturer and a retailer in a manufacturer–retailer channel for a single-period product, in which the manufacturer is dominant. They assumed in the non-cooperative game model that, the manufacturer sets a wholesale price "w/unit" for selling the product to the retailer; given w, the retailer determines: (1) the quantity Q that he ordered from the manufacturer, and (2) the retailer price "p/unit" at which he sold to the consumers. Then they studied how the level of retail-market demand uncertainty would affect the decisions (w, Q, p), the expected manufacturer's and retailer's profits. Li et al. (2002) utilized chance-constrained game theory to investigate the interaction relationship between a manufacturer and a retailer considering the character of demand uncertainty that satisfied a normal distribution. Lau and Lau (2003) studied optimal decisions and outcomes pertaining to a two-echelon supply chain with one manufacturer and multiple retailers form different market/ownership configurations—ranging from a fully vertically integrated supply chain to a situation in which all entities are separately owned and the manufacturer charge different retailers different prices.

It is known that cooperation is an approach of improving competitive advantages of a supply chain (Bylka, 2003; Wang and Benaroch, 2004). This paper extends the research of Lau and Lau (2002) in the context of cooperative game between a manufacturer and a retailer. The non-cooperative game between a manufacturer and a retailer studied by Lau and Lau (2002) is essentially a manufacturer-dominant Stackelberg game. Although the fact that there is a shift of retailing power from manufacturers to retailers, and retailers have equal or even greater power than a manufacturer when it comes to retailing (see, e.g., Dobson and Waterson, 1999), manufacturers still play a leader role in today's China mid-level cars[1] market as it did in China refrigerator and TV markets 15 years ago.

Currently, China's mid-level car market presents the following characteristics. Firstly, sales of mid-level cars maintain high-speed increase, according to the report of the China Automobile News, China's demand for cars is expected to balloon to more than 20 million units by 2020, almost five times of its current output (United Process International, 2004). This situation results in excess demand for the mid-level cars, and let the automakers have the dominance over their franchisors. Secondly, because of the excess demand, automakers like Volkswagen, GM and Honda (arguably the three with the most momentum in China) are finding that their capacity cannot keep up with demand, their cars often take from a week to a month for delivery (Lienert, 2003). Customers usually have to add perquisite on automaker's direction price if they want to receive the cars immediately (Xing, 2003). These market characteristics are similar to the market

[1]According to Forbes (Lienert, 2003), the mid-level cars are defined by the cars like Buick Regal, Honda Accord and Volkswagen Passat, etc., with price RMB ¥150,000–350,000 for the moment.

situations investigated in this paper. Thus, it is still significant to uncover the right way of cooperation for practitioners in current China mid-level car market to improve their benefits. Relative research results will be also helpful for realtor in future's China high-level real estate market.

In this paper, we model the decision-making process of the manufacturer and the retailer as a cooperative game. In the cooperative game, we address the following questions for the supply chain:

(1) How do the manufacturer and the retailer cooperate to improve the supply chain performance?

(2) How do the manufacturer and the retailer make the optimal decision when they cooperate, and how to allocate the overall channel profits?

(3) What are the advantages of cooperation compared with non-cooperation?

(4) How do changes in retail-market demand uncertainty affect the cooperation?

We develop a cooperative mechanism that maximizes the product of the manufacturer's and the retailer's expected profits at an endogenous retail price and at an exogenous retail price, respectively. The cooperative mechanism is essentially a bargaining process (Assaf and Samuel-Cahn, 1998; Osborne and Rubinstein, 1994). The manufacturer and the retailer bargain and make an agreement on per-unit wholesale price, order quantity, and per-unit retail price (when it is determined endogenously). Under such a contract, the manufacturer provides the product at the contract wholesale price, the retailer purchases the product from the manufacturer at the contract order quantity and sells it to customers at the contract retail price. Because of different contributions of different members for the supply chain, the way of allocation of overall channel profits is vital for cooperation, Shapley value is then employed to define the manufacturer's and the retailer's contribution ratios, which are used as the basis of allocating the overall channel profits.

Our research provides a better understanding of the impacts of retail-market demand uncertainty on the cooperative decision-making and the supply chain performance for single-period problems; and by comparing the cooperative results with the non-cooperative ones, our research also provides a better understanding of the cooperation effect.

The paper is organized as follows. Section 2 presents two cooperative game models for a two-echelon supply chain of a single-period product in different market situations. Section 3 studies benefits of cooperation and the cooperative game equilibrium existing condition when wholesale price (w) and the retailer's order quantity (Q) are decision variables; numerical analyses and discussion of the impact of retail-market demand uncertainty are provided in Section 4. Theoretical and numerical analyses of impact of retail-market demand uncertainty when wholesale and retail prices (w and p) as well as order quantity (Q) are decision variables are described in Sections 5 and 6, respectively. Section 7 concludes this paper. All mathematical proofs are found in the appendices.

2. Models and assumptions

Consider a simple two-echelon supply chain consisting of one manufacturer and one retailer, which produces and sells a single-period product. The retail-market demand per period of the single-period product is D, with probability density function (PDF) $f(\cdot)$ and cumulative distribution function (CDF) $F(\cdot)$. Assume that D has a mean value of μ, a standard deviation value of σ, and satisfies uniform distribution supported on $[a, b]$, i.e., $D \sim U[a, b]$. The manufacturer incurs a manufacturing cost of \$$m$ per unit and wholesales the product at \$$w$ per unit to the retailer; the retailer orders Q units of the product from the manufacturer and retails it at price \$$p$ per unit to the consumers. At the end of the period, the retailer's unsold units can be salvaged in an open market for \$$s$ per unit ($p > w > m > s$). The retailer also incurs a loss-of-goodwill cost of \$$t$ per unit for demand not satisfied during the period. Our problem's decision variables are: w, Q or w, Q and p. The manufacturer's and retailer's profits are denoted as θ_{m} and θ_{r}, respectively. Define Θ_{m} and Θ_{r} as the respective expected values of θ_{m} and θ_{r}.

We add subscript "$_{\text{non}}$" or "$_{\text{\_non}}$" to relative variables to differentiate the non-cooperative situation (e.g., Q_{non}, $\Theta_{\text{m\_non}}$) from the cooperative situation (e.g., Q, Θ_{m}), and add superscript "*" to relative variables (e.g., w_{non}, $\Theta_{\text{r\_non}}$) to denote their optimal value (e.g., w^*_{non}, $\Theta^*_{\text{r\_non}}$).

We will explore the impact of retail-market demand uncertainty on the cooperative decisions and cooperation effects compared with non-cooperation in two different situations that reflect various realistic markets. In this paper, cooperation effects involve the overall channel profits, the supply chain members' allocated profits and the supply chain efficiency. The first situation assumes that the wholesale price w and the order quantity Q are decision variables while the retail price p is determined exogenously; the second one sets w, Q and p all be decision variables.

For the sake of comparability of different markets' demand uncertainty, we measure demand uncertainty in its coefficient of variation, which is defined as follows.

Definition 1. If retail-market demand D is a random variable with mean μ and standard deviation σ, then its coefficient of variation c_D is defined as

$$c_D = \frac{\sigma}{\mu}. \tag{1}$$

For the single-period product problem, it is known that (see, e.g., Lau and Lau, 2002; Khouja, 1999):

$$\theta_{\text{m}} = (w - m)Q, \tag{2}$$

$$\theta_{\text{r}} = (p + t - s)\min(D, Q) + (s - w)Q - tD, \tag{3}$$

$$\Theta_{\text{m}} = E(\theta_{\text{m}}) = (w - m)Q, \tag{4}$$

$$\Theta_{\text{r}} = E(\theta_r) = (p + t - s)E(A) + (s - w)Q - tE(D), \tag{5}$$

where $E(A) = E(\min(D, Q)) = \int_{-\infty}^{Q} xf(x)\,\mathrm{d}x + \int_{Q}^{+\infty} Qf(x)\,\mathrm{d}x$.

Cooperation here means that there exists a profit set accepted by the manufacturer and the retailer, if we know a pair of utility functions for the manufacturer and the retailer, the set can be mapped into a subset of two-dimensional Euclidean Space R_2.

We develop a cooperative mechanism that maximizes the product of Θ_{m} and Θ_{r}, through the mechanism, the manufacturer and the retailer can bargain and make an agreement on wholesale price, order quantity, and retail price (when it is determined endogenously). Shapley value is then employed to define the manufacturer's and the retailer's contribution ratios, via which the supply chain members divide out the overall channel profits obtained by their cooperation.

On the assumption that the manufacturer and the retailer are all risk-neutral, we model the cooperative game between the manufacturer and the retailer as follows:

In the first situation, w and Q are decision variables, p is determined exogenously, and the cooperative game model is

$$\underset{(w,Q)}{\text{Max}}\, \Theta_{\text{m}}\Theta_{\text{r}}. \tag{6}$$

Similarly, the cooperative game model of the second situation, where w, Q and p are all decision variables, is

$$\underset{(w,Q,p)}{\text{Max}}\, \Theta_{\text{m}}\Theta_{\text{r}}. \tag{7}$$

The above cooperative game models mean the manufacturer and the retailer have equal status when they make the cooperative decisions. Shapley value is usually defined in coalition game analyses, in this paper, we borrow it as a tool to calculate supply chain members' contribution ratios.

Definition 2. The manufacturer's and the retailer's contribution ratios in the cooperation are defined as follows:

$$l_{\mathrm{m}} = \frac{\varphi_{\mathrm{m}}[v]}{\varphi_{\mathrm{m}}[v] + \varphi_{\mathrm{r}}[v]}, \tag{8}$$

$$l_{\mathrm{r}} = \frac{\varphi_{\mathrm{r}}[v]}{\varphi_{\mathrm{m}}[v] + \varphi_{\mathrm{r}}[v]}, \tag{9}$$

$$\varphi_i[v] = \sum_{T \subseteq N, i \in T} \frac{(|T|-1)!(n-|T|)!}{n!} [v(T) - v(T \backslash \{i\})] \quad \forall i \in N = \{m, r\}, \tag{10}$$

where v is a characteristic function defined on the subsets of N, $v(\phi) = 0$, $v(\{m\}) = \Theta_{\mathrm{m\_non}}$, $v(\{r\}) = \Theta_{\mathrm{r\_non}}$, $v(\{m, r\}) = \Theta_{\mathrm{m}} + \Theta_{\mathrm{r}}$; and $|T|$ denotes the number of the set T, $n = |N|$.

After finding the optimal solutions (w^*, Q^*) and (w^*, Q^*, p^*) to the cooperative game models (6) and (7), respectively, the contribution ratios will be utilized to allocate the overall channel profits, $\Theta^*_{\mathrm{m}} + \Theta^*_{\mathrm{r}}$, between the manufacturer and the retailer.

3. The cooperative situation when *w* and *Q* are decision variables

Firstly, we investigate the situation when w and Q are decision variables while p is determined exogenously. The manufacturer's and the retailer's expected profits are, respectively,

$$\Theta_{\mathrm{m}} = (w - m)Q, \tag{11}$$

$$\Theta_{\mathrm{r}} = (p + t - s)E(A) + (s - w)Q - t\mu. \tag{12}$$

Because $D \sim U[a, b]$, from the property of the uniform distribution it is known that

$$a = \mu - \sqrt{3}\sigma, \quad b = \mu + \sqrt{3}\sigma, \tag{13}$$

$$F^{-1}(x) = \mu - \sqrt{3}\sigma + 2\sqrt{3}\sigma x. \tag{14}$$

We summarize the results of the non-cooperative game investigated by Lau and Lau (2002) as follows:

$$w^*_{\mathrm{non}} = m + \frac{(p + t - s)\mu + \sqrt{3}\sigma(p + t + s - 2m)}{4\sqrt{3}\sigma}, \tag{15}$$

$$Q^*_{\mathrm{non}} = \frac{1}{2}\left[\mu + \frac{\sqrt{3}\sigma(p + t + s - 2m)}{p + t - s}\right], \tag{16}$$

$$\Theta^*_{\mathrm{m\_non}} = \frac{[(p + t - s)\mu + \sqrt{3}\sigma(p + t + s - 2m)]^2}{8\sqrt{3}\sigma(p + t - s)}, \tag{17}$$

$$\Theta^*_{\mathrm{r\_non}} = \frac{[(p + t - s)\mu + \sqrt{3}\sigma(p + t + s - 2m)]^2 - [2(p + t - s)(\mu - \sqrt{3}\sigma)]^2}{16\sqrt{3}\sigma(p + t - s)} - t\mu. \tag{18}$$

Let Q^*_{c} and Θ^*_{c} denote the respective optimal order quantity and the expected profit of supply chain when the supply chain is vertically integrated. Following Lau and Lau (2002), we define the supply chain efficiency as follows:

Definition 3. Let the supply chain efficiency be defined by

$$E_f = \frac{\Theta^*_{\rm m} + \Theta^*_{\rm r}}{\Theta^*_{\rm c}}. \tag{19}$$

The following proposition implies that a decentralized supply chain can obtain higher channel profits when the members cooperate than that when they do not cooperate, and even can reach the same profits of a vertically integrated supply chain.

Proposition 1. *If the cooperative game model* (6) *exists the equilibrium solutions, then the optimal order quantity* Q^* *equals* $Q^*_{\rm c}$, *and* $\Theta^*_{\rm m} + \Theta^*_{\rm r}$ *equals* $\Theta^*_{\rm c}$.

From Proposition 1 and Definition 3, we have the following corollary:

Corollary 1. *If the cooperative game model* (6) *exists the equilibrium solutions, the supply chain efficiency* E_f *is constantly equal to* 1.

Lau and Lau's (2002) investigation indicates that the supply chain efficiency is always less than 1 at non-cooperative situation. Corollary 1 implies that the supply chain efficiency and the overall channel profits are increased by supply chain members' cooperation, i.e., the overall channel performance is improved by the cooperation. These results have also been recognized in many real business situations (see, e.g., Kumar, 1996; Li et al., 2002).

Let $\Pi^*_{\rm m}$ and $\Pi^*_{\rm r}$ denote the manufacturer's and the retailer's allocated profits in the cooperative situation, respectively. The overall channel profits are allocated via the manufacturer's and the retailer's contribution ratios defined in Definition 2. In general, neither the manufacturer nor the retailer would be willing to accept less profit at cooperation than at non-cooperation. We have the following proposition to show the basis of the cooperation:

Proposition 2. *If the cooperative game model* (6) *exists the equilibrium solutions, then the manufacturer's and the retailer's allocated profits at cooperation are more than those obtained at non-cooperation, respectively,* i.e., $\Pi^*_{\rm m} > \Theta^*_{\rm m\_non}$ and $\Pi^*_{\rm r} > \Theta^*_{\rm r\_non}$. *The manufacturer's increased profit equals to the retailer's increased profit at cooperation compared with non-cooperation,* i.e., $\Pi^*_{\rm m} - \Theta^*_{\rm m\_non} = \Pi^*_{\rm r} - \Theta^*_{\rm r\_non}$.

Proposition 2 implicates that the contribution ratios defined by (8) and (9) is reasonable in allocating the overall channel profits, the manufacturer and the retailer have equal profit gain at cooperation compared with non-cooperation.

Although cooperation can improve overall channel profits, the following theorem indicates that cooperation between the manufacturer and the retailer can be implemented only when retail-market demand is relatively steady.

Theorem 1. *In the cooperative game model* (6), *when retail-market demand D is uniformly distributed,* i.e., $D \sim U[a,b]$, *the cooperative game between the manufacturer and the retailer exists a unique equilibrium solution* (w^*, Q^*) *if and only if the coefficient of variation* c_D *satisfies* $0 \leqslant c_D < c_D^{\max}$, *where*

$$c_D^{\max} = \min\left\{\frac{1}{\sqrt{3}}, \frac{(p+t-s)(p-m)}{\sqrt{3}(p+t-m)(m-s)}\right\}, \tag{20}$$

$$Q^* = \mu + \frac{\sqrt{3}(p+t+s-2m)\sigma}{p+t-s}, \tag{21}$$

$$w^* = \frac{(p+m)(p+t-s)\mu + \sqrt{3}[m(p+t-3m) + s(p+t+m)]\sigma}{2(p+t-s)\mu + 2\sqrt{3}(p+t+s-2m)\sigma}. \tag{22}$$

Theorem 1 implies that only when retail-market demand changes smoothly, i.e., the coefficient of variation of retail-market demand is sufficiently small and does not exceed an upper bound, the manufacturer and the retailer can bargain to induce an acceptable contract, and maintain the cooperative relationship.

Substituting (w^*, Q^*) of (21) and (22) into (11) and (12), we can write the manufacturer's and the retailer's expected profits as

$$\Theta_{\mathrm{m}}^* = \Theta_{\mathrm{r}}^* = \frac{1}{2}\left[(p-m)\mu - \frac{\sqrt{3}(p+t-m)(m-s)\sigma}{p+t-s}\right]. \tag{23}$$

So the optimal overall channel profits are

$$\Theta_{\mathrm{m}}^* + \Theta_{\mathrm{r}}^* = (p-m)\mu - \frac{\sqrt{3}(p+t-m)(m-s)\sigma}{p+t-s}. \tag{24}$$

There are similar results to Theorem 1 when retail-market demand satisfies a normal distribution.

Corollary 2. *In the cooperative game model* (6), *when retail-market demand D is normally distributed, i.e., $D \sim N(\mu, \sigma^2)$, the cooperative game between the manufacturer and the retailer exists the optimal equilibrium solutions (w^*, Q^*) if and only if the coefficient of variation c_D satisfies $0 \leqslant c_D < c_D^{\max}$, where*

$$c_D^{\max} = \min\left\{\frac{1}{\Phi^{-1}(\beta)}, \frac{\sqrt{2\pi}(p-m)}{p+t-s}\mathrm{e}^{c^2}\right\},$$

$$c = Erf\left[0, \frac{p+t+s-2m}{p+t-s}\right], \quad Erf[0,x] = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-z^2}\,\mathrm{d}z,$$

parameter β is a confidence coefficient that for non-negative demand.

Similar results to Corollary 2 can be obtained when D satisfies a Beta distribution or other common distributions in a similar way. We leave it to the readers considering the length limitation of the paper.

The following proposition shows the way how retail-market demand uncertainty impacts the optimal decisions and the members' allocated profits for a cooperative supply chain.

Proposition 3. *If the cooperative game model* (6) *exists the equilibrium solutions and $D \sim U[a, b]$, then*

(1) *Q^* may be increasing, decreasing, or constant in c_D, which depends on the parameters p, m, s, and t; w^* is decreasing in c_D.*

(2) *The overall channel profits, $\Theta_{\mathrm{m}}^* + \Theta_{\mathrm{r}}^*$, are decreasing in c_D.*

(3) *Π_{m}^* is decreasing in c_D.*

(4) *The retailer achieves her maximum profit Π_{r}^* at $c_D = c_D^*$, where c_D^* is uniquely determined by*

$$c_D^* = \frac{\sqrt{5}(p+t-s)}{\sqrt{3[5(p+t-s)^2 + 12(p+t-m)(m-s)]}}.$$

In the above proposition, the conclusions that the optimal wholesale price, the overall channel profits and the manufacturer's allocated profit decrease in retail-market uncertainty are rather intuitive. However, impacts of retail-market demand uncertainty on the optimal order quantity and the retailer's allocated profit are counterintuitive. These are because, if the retail price is sufficiently high, the retailer is delight to order more from the manufacturer to increase her profit; but if retail-market demand varies too acutely, the retailer would not earn more from ordering more products.

4. Numerical analysis of impact of c_D when w and Q are decision variables

In this section, we give some numerical illustrations to explore impacts of retail-market demand uncertainty on the optimal decisions and supply chain performance. We perform it in two parts, one is at a given retail price level, the other is at different retail price levels.

Without loss of generality, let $m = 1$, $t = 0$, $s = 0$, $\mu = 100$ and $p = 1.5$. According to Theorem 1, it is known that the cooperation game exists equilibrium solutions if and only if the coefficient of variation of retail-market demand is less than 0.577. Let $c_D \in [0.15, 0.55]$, numerical results are calculated and shown in Table 1.

One issue should be illuminated in Table 1. In the non-cooperation situation, $w^*_{\text{non}} > p$ when $c_D < 0.346$ (cf. (15)). We set $w^*_{\text{non}} = p = 1.5$ in Table 1 as Lau and Lau (2002) did (it corresponds to the bold face in Table 1). Although there is no such case at the cooperative game model, for the convenience of comparing the cooperative situation with the non-cooperative situation, we ignore cases of $w^*_{\text{non}} > p$ hereafter.

Table 1
The optimal solutions when Q and w are decision variables with demand satisfying uniform distribution. $m = 1$, $s = 0$, $t = 0$, $\mu = 100$, $c_D = 0.15$–0.55

c_D	w^*_{non}	w^*	Q^*_{non}	Q^*	$\Theta^*_{\text{m\_non}}$	$\Theta^*_{\text{r\_non}}$	$\Theta^*_{\text{m\_non}} + \Theta^*_{\text{r\_non}}$	$\Theta^*_{\text{m}} + \Theta^*_{\text{r}}$	$E^*_{f\text{\_non}}$	l_{m}	l_{r}	Π^*_{m}	Π^*_{r}
$p = 1.5$													
0.15	**1.500**	1.226	74.02	91.34	37.00	0.00	37.01	41.34	0.90	0.95	0.05	39.27	2.07
0.25	**1.500**	1.208	56.70	85.57	28.35	0.00	28.35	35.57	0.80	0.90	0.10	32.01	3.56
0.35	1.494	1.187	39.90	79.79	19.69	0.25	19.95	29.79	0.67	0.83	0.17	24.73	5.06
0.40	1.416	1.175	38.45	76.91	16.01	2.90	18.91	26.91	0.70	0.74	0.26	19.91	6.70
0.45	1.356	1.162	37.01	74.02	13.18	4.25	17.43	24.02	0.73	0.69	0.31	16.57	7.45
0.50	1.308	1.149	35.57	71.13	10.95	4.70	15.66	21.13	0.74	0.65	0.35	13.73	7.40
0.55	1.269	1.134	34.12	68.25	9.17	4.50	13.66	18.25	0.75	0.63	0.37	11.50	6.75
$p = 2.0$													
0.15	**2.000**	1.435	74.02	100.00	74.02	0.00	74.02	87.01	0.85	0.93	0.07	80.92	6.09
0.25	**2.000**	1.392	56.70	100.00	56.70	0.00	56.70	78.35	0.72	0.86	0.14	67.38	10.97
0.35	1.825	1.349	50.00	100.00	41.24	7.83	49.07	69.69	0.70	0.74	0.26	51.57	18.12
0.40	1.722	1.327	50.00	100.00	36.08	11.23	47.31	65.36	0.72	0.69	0.31	45.10	20.26
0.45	1.642	1.305	50.00	100.00	32.08	12.92	44.99	61.03	0.74	0.66	0.34	40.28	20.75
0.50	1.577	1.284	50.00	100.00	28.87	13.40	42.26	56.70	0.75	0.64	0.36	36.29	20.41
0.55	1.525	1.262	50.00	100.00	26.24	13.01	39.26	52.37	0.75	0.63	0.37	32.99	19.38
$p = 3.0$													
0.15	**3.000**	1.841	74.02	108.66	148.04	0.00	148.04	182.68	0.81	0.91	0.09	166.24	16.44
0.25	2.982	1.748	57.22	114.43	113.41	1.02	114.43	171.13	0.67	0.83	0.17	142.04	29.09
0.35	2.487	1.664	60.10	120.21	89.38	25.51	114.89	159.59	0.72	0.70	0.30	111.71	47.88
0.40	2.333	1.625	61.55	123.09	82.01	30.79	112.80	153.81	0.73	0.67	0.33	103.05	50.76
0.45	2.212	1.588	62.99	125.98	76.36	33.50	109.86	148.04	0.74	0.64	0.36	94.75	53.29
0.50	2.116	1.552	64.43	128.87	71.91	34.40	106.31	142.27	0.75	0.63	0.37	89.63	52.64
0.55	2.037	1.518	65.88	131.75	68.33	33.99	102.33	136.49	0.75	0.63	0.37	85.99	50.50

At a given retail price level, we can confirm impacts of c_D on the cooperation decisions and cooperation effect from the propositions obtained analytically for the first cooperative situation in Section 3, so we omit detailed analysis for the given retail price level.

In order to analyze impacts of retail-market demand uncertainty at different retail price levels, we provide numerical results when p equals to 2.0 and 3.0, as shown in Table 1. By comparing results of different retail price levels, we can observe that, a higher retail price would result in a higher wholesale price, a higher order quantity, and higher profits for the manufacturer and the retailer, and impacts of retail-market demand uncertainty at different retail price levels are similar on the whole.

Under a higher retail price level, w^* decreases faster in the increase of c_D than that under a lower retail price level, this means that retail-market demand uncertainty has a larger effect on the optimal wholesale price at a higher retail price level.

From Table 1, when c_D increases from 0.35 to 0.55, the overall channel profits decrease 38.74%, 24.85% and 14.47%, at the retail price $p = 1.5$, $p = 2.0$ and $p = 3.0$, respectively. This means that retail-market demand uncertainty has less influence on the overall channel profits at a higher retail price level than that at a lower retail price level.

5. The cooperative situation when w and p as well as Q are decision variables

In this section, we explore another market situation when the wholesale price, the retail price and the order quantity are all decision variables.

When the retail price is a decision variable, there exists a specification of a price–demand relationship, a popular price–demand relationship is

$$\mu = K - \alpha p, \tag{25}$$

where K relates to retail-market size, "α" in the above relationship relates to μ's price sensitivity (see, Lau and Lau, 2002).

Suppose retail-market demand is also uniformly distributed, i.e., $D \sim U[a, b]$, then from the property of the uniform distribution, it is known that

$$a = \mu - \sqrt{3}\sigma = K - \alpha p - \sqrt{3}\sigma, \quad b = \mu + \sqrt{3}\sigma = K - \alpha p + \sqrt{3}\sigma. \tag{26}$$

The inverse function of CDF of retail-market demand is

$$F^{-1}(x) = K - \alpha p - \sqrt{3}\sigma + 2\sqrt{3}\sigma x, \quad x \in [0, 1]. \tag{27}$$

Based on the description of the supply chain framework in Section 2, the manufacturer's and the retailer's expected profits are as follows:

$$\Theta_{\mathrm{m}} = (w - m)Q, \tag{28}$$

$$\Theta_{\mathrm{r}} = (p + t - s)E(A) + (s - w)Q - t(K - \alpha p). \tag{29}$$

By analyzing the cooperative game model (7), we have the following lemma:

Lemma 1. *The possible equilibrium solutions* (w^*, Q^*, p^*) *for the cooperative game model* (7) *are determined as follows*:

$$p^* = s - t + (s - m)/y, \tag{30}$$

$$Q^* = K - \alpha p^* - \sqrt{3}\sigma + 2\sqrt{3}\sigma(y + 1), \tag{31}$$

$$w^* = \frac{1}{2Q^*}\Big[(p^*+t-s)\Big(Q^*-\sqrt{3}\sigma(y+1)^2\Big)+(s+m)Q^*-t(K-\alpha p^*)\Big], \quad (32)$$

y in (30)–(32) *satisfies*

$$y^3 + Uy + V = 0, \quad (33)$$

where

$$U = -\frac{K+\alpha m+2\alpha(t-s)}{\sqrt{3}\sigma},$$

$$V = \frac{2\alpha(s-m)}{\sqrt{3}\sigma}.$$

The results of Lemma 1 are necessary in proof of Theorem 2. Theorem 2 indicates that cooperation between the manufacturer and the retailer is also conditional on retail-market demand uncertainty in the second cooperative situation.

Theorem 2. *In the cooperative game model* (7), *when retail-market demand D is uniformly distributed, i.e.,* $D \sim U[a,b]$, *the cooperative game between the manufacturer and the retailer exists a unique equilibrium solution* (w^*, Q^*, p^*) *if and only if the coefficient of variation* c_D *satisfies* $0 \leqslant c_D < c_D^{\max}$, *where*

$$c_D^{\max} = \min\left\{\frac{1}{\sqrt{3}}, \frac{(17K+\alpha(16t-m-16s))\sqrt{(-K+\alpha m)(-17K+\alpha(-16t+m+16s))}}{16\sqrt{3}\alpha^2(m-s)^2} - \frac{[-71K^2+2\alpha K(-52t+19m+52s)+\alpha^2(-32t^2+40tm+m^2+64ts-40ms-32s^2)]}{16\sqrt{3}\alpha^2(m-s)^2}\right\}, \quad (34)$$

w^*, Q^* *and* p^* *are determined by* (30)–(32).

The results in Theorem 2 are similar to those in Theorem 1, which are described for the first cooperative situation. Theorem 2 implies that only when retail-market demand changes smoothly, i.e., the coefficient of variation of retail-market demand is sufficiently small and does not exceed an upper bound, the manufacturer and the retailer can cooperate to induce an acceptable contract, and maintain their cooperative relationship.

Unlike the case of the first cooperative game model, because of the complexity of equilibrium solutions of cooperative game model (7), it is impossible to derive closed-form solutions for the overall channel profits, the manufacturer's and the retailer's allocated profits, the supply chain efficiency, and impacts of retail-marker demand uncertainty on the optimal decisions and the profits in cooperative and non-cooperative situations (the complexity of non-cooperative situation has been confirmed by Lau and Lau, 2002). We thus use numerical examples to study the above issues for the second cooperative situation in the next section.

6. Numerical analysis of c_D's impact when w and p as well as Q are decision variables

Without loss of generality, let $m = 1$, $t = 0$, $s = 0$ and $K = 100$, the numerical solutions to the cooperative game model (7) can be obtained for different α-values, the overall channel profits, the manufacturer's and the retailer's allocated profits, and the supply chain efficiency can then be calculated at equilibrium solutions (w^*, Q^*, p^*).

According to Theorem 2, if $\alpha = 5.0$, the condition that cooperative game between the manufacturer and the retailer exists a unique optimal equilibrium solution is that c_D does not exceed an upper bound of 0.577.

Table 2
Optimal solutions when w, Q, and p are decision variables with uniform demand distribution (for the results of non-cooperative situation, please see Lau and Lau, 2002)

σ	w^*_{non}	p^*_{non}	Q^*_{non}	$\Theta^*_{\text{m\_non}}$	$\Theta^*_{\text{r\_non}}$	$\Theta^*_{\text{m\_non}}+\Theta^*_{\text{r\_non}}$	$E_{f\text{\_non}}$	w^*	p^*	Q^*	$\Theta^*_{\text{m}}+\Theta^*_{\text{r}}$	Q^*_{c}	Θ^*_{c}	l_{m}	l_{r}	Π^*_{m}	Π^*_{r}
$\alpha = 10.0$																	
5	9.33	14.3	25.86	215.34	113.64	328.98	0.74	5.0	10.49	54.5	443.41	54.5	443.4	0.61	0.38	272.5	170.8
10	8.31	13.5	28.52	208.33	113.46	321.79	0.74	4.5	10.48	61.6	435.58	61.6	435.5	0.60	0.39	265.2	170.3
15	7.42	12.8	31.74	203.81	112.60	316.41	0.74	4.1	10.47	68.6	427.75	68.6	427.7	0.60	0.39	259.4	168.2
20	6.78	12.3	34.90	201.51	107.30	308.81	0.74	3.7	10.46	75.6	419.91	75.6	419.9	0.61	0.38	257.0	162.8
$\alpha = 10.0$																	
5	4.94	7.27	24.20	95.30	49.94	145.24	0.74	2.9	5.486	50.6	195.42	50.6	195.4	0.61	0.38	120.3	75.03
10	4.47	6.87	26.10	90.54	48.11	138.65	0.74	2.6	5.471	56.2	188.34	56.2	188.3	0.61	0.38	115.39	72.95
15	4.00	6.51	28.92	86.87	47.44	134.31	0.74	2.4	5.456	61.8	181.26	61.8	181.2	0.60	0.39	110.35	70.91
20	3.66	6.23	31.59	84.13	44.48	128.61	0.74	2.2	5.442	67.4	174.19	67.4	174.1	0.61	0.38	106.9	67.27
$\alpha = 50.0$																	
3	1.45	1.69	12.01	5.37	2.68	8.05	0.75	1.2	1.476	24.3	10.80	24.3	10.80	0.62	0.37	6.75	4.05
6	1.39	1.62	11.51	4.53	2.27	6.80	0.74	1.1	1.451	23.5	9.15	23.5	9.15	0.62	0.37	5.71	3.44
9	1.34	1.55	11.08	3.74	1.92	5.66	0.75	1.1	1.423	22.5	7.57	22.5	7.57	0.62	0.38	4.70	2.87
12	1.28	1.49	10.51	2.99	1.56	4.55	0.75	1.1	1.139	21.3	6.06	21.3	6.06	0.61	0.38	3.75	2.31
15	1.24	1.42	9.68	2.29	1.17	3.46	0.75	1.1	1.360	19.7	4.64	19.7	4.64	0.62	0.37	2.88	1.76
18	1.19	1.35	8.84	1.65	0.83	2.48	0.75	1.0	1.322	17.9	3.32	17.9	3.32	0.62	0.37	2.07	1.25

For the convenience of comparing impacts of demand uncertainty in the cooperative situation with those in the non-cooperative situation investigated by Lau and Lau (2002), we substitute σ for c_D, and relative results are depicted in Table 2.

Table 2 indicates that impacts of retail-market demand uncertainty on members' profits ($\Pi^*_{\text{m}}, \Pi^*_{\text{r}}$) in the cooperative situation are similar to those in the non-cooperative ($\Theta^*_{\text{m\_non}}, \Theta^*_{\text{r\_non}}$), which decrease as σ increases. w^*, p^* and $\Theta^*_{\text{m}}+\Theta^*_{\text{r}}$ also decrease as σ increases. Q^* increases as σ increases, however, when α is sufficiently big (e.g. $\alpha = 50.0$), it decreases as σ increases. This indicates that, in the second cooperative situation, the properties about impacts of retail-market uncertainty on the optimal wholesale price, the overall channel profits and the manufacturer's allocated profit are similar to those of Proposition 3 obtained analytically for the first cooperative game model. Π^*_{r} always decreases in σ in the second cooperative situation while it is a unimodal function of σ in the first cooperative situation.

It can also be observed that Q^* and p^* equal Q^*_{c} and p^*_{c}, respectively. The overall channel profits at cooperation equal to the channel profits of the vertically integrated supply chain, i.e., $\Theta^*_{\text{m}}+\Theta^*_{\text{r}} = \Theta^*_{\text{c}}$, and the supply chain efficiency is constant with $E_f = 1$. $E_f > E_{f\text{\_non}}$, $\Theta^*_{\text{m}}+\Theta^*_{\text{r}} > \Theta^*_{\text{m\_non}}+\Theta^*_{\text{r\_non}}$, $\Pi^*_{\text{m}} > \Theta^*_{\text{m\_non}}$, $\Pi^*_{\text{r}} > \Theta^*_{\text{r\_non}}$ and $\Pi^*_{\text{m}} - \Theta^*_{\text{m\_non}} = \Pi^*_{\text{r}} - \Theta^*_{\text{r\_non}}$ are also true in Table 2. These results imply that the properties of cooperation effect for the second cooperative situation are similar to those in Propositions 1 and 2 for the first cooperative situation.

Compared with the non-cooperative situation, the cooperation leads to lower wholesale and retail prices. Thus, not only the overall channel can benefit from the cooperation, but also the customer can benefit from the cooperation. In current China mid-level car market, car franchisors usually raise retail price for their own benefit, the above results can provide guidelines for the automaker and her franchisors.

7. Conclusion

We studied the cooperation mechanism in a manufacturer-retailer supply chain, the manufacturer and the retailer bargain and make an agreement on wholesale price, order quantity and retail price (when it is decision variable). Under such an agreement, when retail-market demand satisfies uniform distribution, our research results show that, cooperation defined in this paper yields a lower wholesale price, a lower retail price (when it is decision variable), a higher supply chain efficiency and higher overall channel profits. The overall channel profits can be divided out among supply chain members via their contribution ratios. The contribution ratios defined in this paper is reasonable, because according to the contribution ratios, channel members' allocated profits in the cooperative situations have equal additions compared with those in the non-cooperative situations.

Our research results also indicate that, the described cooperation is conditional on retail-market demand uncertainty: it can be implemented if, and only if, the fluctuation of retail-market demand is relatively small and coefficient of variation of retail-market demand does not exceed an upper bound. Impacts of retail-market demand uncertainty on wholesale price, order quantity and/or retail price have also been investigated through analytical and numerical analyses.

Our study is based on the manufacturer as the leader in the non-cooperative situation, which is still significant on providing guidelines for practitioners in current China mid-level car market that is similar to situations described in the paper.

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Appendix A

Proof of Proposition 1. If the cooperative game model (6) exists the optimal equilibrium solutions (w^*, Q^*), and from the first order condition, (w^*, Q^*) are determined by

$$Q^* = F^{-1}\left(\frac{p+t-m}{p+t-s}\right), \quad w^* = \frac{1}{2Q^*}\{(p+t-s)E(A^*) + (s+m)Q^* - t\mu\},$$

$$\Theta^*_{\mathrm{m}} + \Theta^*_{\mathrm{r}} = (p+t-s)E(A^*) + (s-m)Q^* - t\mu. \tag{35}$$

Denote $A_{\mathrm{c}} = \min(D, Q_{\mathrm{c}})$, $A^*_{\mathrm{c}} = \min(D, Q^*_{\mathrm{c}})$. It is known that

$$\Theta_{\mathrm{c}} = (p+t-s)E(A_{\mathrm{c}}) + (s-m)Q_{\mathrm{c}} - t\mu. \tag{36}$$

Eq. (36) gives $Q^*_{\mathrm{c}} = F^{-1}\left(\frac{p+t-m}{p+t-s}\right) = Q^*$. Hence $A^*_{\mathrm{c}} = A^*$.

$$\Theta^*_{\mathrm{c}} = (p+t-s)E(A^*_{\mathrm{c}}) + (s-m)Q^*_{\mathrm{c}} - t\mu = \Theta^*_{\mathrm{m}} + \Theta^*_{\mathrm{r}}. \quad \square \tag{37}$$

Proof of Proposition 2. Form Definition 2, we have Shapley value as follows:

$$\varphi_{\mathrm{m}} = (\Theta^*_{\mathrm{m}} + \Theta^*_{\mathrm{r}} - \Theta^*_{\mathrm{r\_non}} + \Theta^*_{\mathrm{m\_non}})/2, \quad \varphi_{\mathrm{r}} = (\Theta^*_{\mathrm{m}} + \Theta^*_{\mathrm{r}} - \Theta^*_{\mathrm{m\_non}} + \Theta^*_{\mathrm{r\_non}})/2.$$

Then the manufacturer's and the retailer's contribution ratios are, respectively,

$$l_{\rm m}=\frac{\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm r\_non}+\Theta^*_{\rm m\_non}}{2(\Theta^*_{\rm m}+\Theta^*_{\rm r})},\quad l_{\rm r}=\frac{\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm m\_non}+\Theta^*_{\rm r\_non}}{2(\Theta^*_{\rm m}+\Theta^*_{\rm r})}.$$

Then, the manufacturer's and the retailer's allocated profits are, respectively,

$$\Pi^*_{\rm m}=\frac{\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm r\_non}+\Theta^*_{\rm m\_non}}{2},\quad \Pi^*_{\rm r}=\frac{\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm m\_non}+\Theta^*_{\rm r\_non}}{2}.$$

Thus,

$$\Pi^*_{\rm m}-\Theta^*_{\rm m\_non}=\Pi^*_{\rm r}-\Theta^*_{\rm r\_non}=\frac{(\Theta^*_{\rm m}+\Theta^*_{\rm m})-(\Theta^*_{\rm r\_non}+\Theta^*_{\rm m\_non})}{2}.$$

From Proposition 1 and Corollary 1, we have $\Theta^*_{\rm m}+\Theta^*_{\rm m}>\Theta^*_{\rm r\_non}+\Theta^*_{\rm m\_non}$, then

$$\Pi^*_{\rm m}-\Theta^*_{\rm m\_non}=\Pi^*_{\rm r}-\Theta^*_{\rm r\_non}>0,\quad \text{i.e.,}\quad \Pi^*_{\rm m}>\Theta^*_{\rm m\_non},\ \Pi^*_{\rm r}>\Theta^*_{\rm r\_non}.\quad \square$$

Proof of Theorem 1. From the first order conditions of the cooperative game model (6), we have

$$Q^*=F^{-1}\left(\frac{p+t-m}{p+t-s}\right)=\mu+\frac{\sqrt{3}(p+t+s-2m)\sigma}{p+t-s}, \tag{38}$$

$$w^*=\frac{(p+m)(p+t-s)\mu+\sqrt{3}[m(p+t-3m)+s(p+t+m)]\sigma}{2(p+t-s)\mu+2\sqrt{3}(p+t+s-2m)\sigma}. \tag{39}$$

The second order condition is the sufficient condition of existing equilibrium solutions. According to Gottfried and Weisman (1973), the second order condition of cooperative game model (6) is equivalent to

$$\left.\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w^2}\right|_{(w^*,Q^*)}<0,\quad \left.\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w^2}\cdot\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial Q^2}-\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w\partial Q}\cdot\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial Q\partial w}\right|_{(w^*,Q^*)}>0.$$

Since

$$\left.\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w^2}\right|_{(w^*,Q^*)}=-2(Q^*)^2<0, \tag{40}$$

$$\left.\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w^2}\cdot\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial Q^2}-\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial w\partial Q}\cdot\frac{\partial^2(\Theta_{\rm m}\Theta_{\rm r})}{\partial Q\partial w}\right|_{(w^*,Q^*)}$$
$$=M[(p+t-s)(p-m)\mu-\sqrt{3}(p+t-m)(m-s)\sigma]. \tag{41}$$

In (41), $M=\dfrac{[(p+t-s)\mu+\sqrt{3}(p+t+s-2m)\sigma]^2}{2\sqrt{3}(p+t-s)^2\sigma}$ is always positive.

So the second order condition equals

$$(p+t-s)(p-m)\mu-\sqrt{3}(p+t-m)(m-s)\sigma>0, \tag{42}$$

$$c_D<\frac{(p+t-s)(p-m)}{\sqrt{3}(p+t-m)(m-s)}. \tag{43}$$

Because $D \sim U[a,b]$, in general, $a = \mu - \sqrt{3}\sigma > 0$, so $c_D < 1/\sqrt{3}$. Combine this with (43), we obtain

$$0 \leqslant c_D < c_D^{\max} = \min\left\{\frac{1}{\sqrt{3}}, \frac{(p+t-s)(p-m)}{\sqrt{3}(p+t-m)(m-s)}\right\}. \quad \square$$

Proof of Corollary 2. For $D \sim N(\mu, \sigma^2)$,

$$E(A^*) = \int_{-\infty}^{Q^*} xf(x)\,\mathrm{d}x + \int_{Q^*}^{+\infty} Q^* f(x)\,\mathrm{d}x = -\sigma^2 f(Q^*) + \mu F(Q^*) + Q^*[1 - F(Q^*)]. \tag{44}$$

From the first order condition, we have

$$Q^* = F^{-1}\left(\frac{p+t-m}{p+t-s}\right), \quad w^* = \frac{1}{2Q^*}\{(p+t-s)E(A^*) + (s+m)Q^* - t\mu\}.$$

The CDF of standard normal distribution can be written as

$$\Phi(x) = \frac{1}{2}\left(1 + Erf\left[0, \frac{x}{\sqrt{2}}\right]\right), \text{ where } Erf[0,x] = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-z^2}\,\mathrm{d}z.$$

Then, $\Phi^{-1}(x) = \sqrt{2}Erf^{-1}[0, 2x-1]$, and because $D \sim N(\mu, \sigma^2)$, so we have

$$F(x) = \Phi((x-\mu)/\sigma), \quad F^{-1}(x) = \mu + \sigma \cdot \Phi^{-1}(x) = \mu + \sqrt{2}\sigma \cdot Erf^{-1}[0, 2x-1].$$

Hence,

$$Q^* = F^{-1}\left(\frac{p+t-m}{p+t-s}\right) = \mu + \sqrt{2}\sigma \cdot c,\ w^* = \frac{[-\sigma^2(p+t-s)f(Q^*) + (p-m)\mu]}{2Q^*}, \text{where } c = Erf\left[0, \frac{p+t+s-2m}{p+t-s}\right].$$

From the second order condition, we can obtain

$$(p-m)\mu > \sigma^2(p+t-s)f(Q^*). \tag{45}$$

Combining the PDF of normal distribution and (45) gives

$$f(Q^*) = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(Q^*-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-c^2}. \tag{46}$$

Substituting (46) into (45) gives

$$c_D < \frac{\sqrt{2\pi}(p-m)}{p+t-s}\mathrm{e}^{c^2}. \tag{47}$$

At a given confidence coefficient β, $P(D \geqslant 0) \geqslant \beta$ yields $\mu/\sigma \geqslant \Phi^{-1}(\beta)$, or $c_D \leqslant 1/\Phi^{-1}(\beta)$, combining it with (47) gives $0 \leqslant c_D < c_D^{\max}$, where $c_D^{\max} = \min\left\{\frac{1}{\Phi^{-1}(\beta)}, \frac{\sqrt{2\pi}(p-m)}{p+t-s}\mathrm{e}^{c^2}\right\}$. $\square$

Proof of Proposition 3. *Part* (1). From (21) and Definition 1, we have that $\frac{\partial Q^*}{\partial c_D} = \frac{\sqrt{3}\mu(p+t+s-2m)}{p+t-s}$, its sign is determined by $p+t+s-2m$ and can be either positive, zero or negative. From (22) and Definition 1, we also have $\frac{\partial w^*}{\partial c_D} = -\frac{\sqrt{3}(p+t-s)[(p-m)^2 + (p-s)t]}{2[p+t-s+\sqrt{3}c_D(p+t+s-2m)]} < 0$.

Part (2). From (24), it is obvious that $\dfrac{\partial(\Theta^*_{\rm m}+\Theta^*_{\rm r})}{\partial c_D} = -\dfrac{\sqrt{3}\mu(p+t-m)(m-s)}{p+t-s} < 0$.

Part (3). From the proof of Proposition 2, we have $\Pi^*_{\rm m} = (\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm r\_non}+\Theta^*_{\rm m\_non})/2$ and

$$\frac{\partial \Pi^*_{\rm m}}{\partial c_D} = \frac{5[3(p+t+s-2m)^2c_D^2-(p+t-s)^2]}{32\sqrt{3}(p+t-s)c_D^2}.$$

From Theorem 1, $c_D < c_D^{\max} \leqslant 1/\sqrt{3}$, thus $\dfrac{\partial \Pi^*_{\rm m}}{\partial c_D} < \dfrac{5(p+t-m)(s-m)}{8\sqrt{3}(p+t-s)c_D^2} < 0$.

Part (4). From the proof of Proposition 2, we have $\Pi^*_{\rm r} = (\Theta^*_{\rm m}+\Theta^*_{\rm r}-\Theta^*_{\rm m\_non}+\Theta^*_{\rm r\_non})/2$ and

$$\frac{\partial \Pi^*_{\rm r}}{\partial c_D} = \frac{5(p+t-s)^2-3[5(p+t-s)^2+12(p+t-m)(m-s)]c_D^2}{32\sqrt{3}c_D^2}.$$

Let $G(c_D) = 5(p+t-s)^2-3[5(p+t-s)^2+12(p+t-m)(m-s)]c_D^2$.

We know that $0 \leqslant c_D < c_D^{\max}$, since $G(c_D)$ is continuous and $G(0)>0$, $G(c_D^{\max})<0$. Because $\partial^2\Pi^*_{\rm r}/\partial c_D^2 = -5(p+t-s)/16\sqrt{3}c_D^3 < 0$ and $\partial^2\Pi^*_{\rm r}/\partial c_D^2<0$ at $\partial\Pi^*_{\rm r}/\partial c_D = 0$, which implies that $\partial\Pi^*_{\rm r}/\partial c_D$ is a unimodal function. Conjunction with $G(0)>0$, $G(c_D^{\max})<0$, guarantees the uniqueness of c_D^* and $0 \leqslant c_D^* < c_D^{\max}$.

The expression of c_D^* can be obtained by directly solving the equation $G(c_D)=0$. □

Appendix B

Proof of Lemma 1. From the first order condition of the cooperative game model (7), $F(Q^*) = (p^*+t-m)/(p^*+t-s)$, then

$$Q^* = a+(b-a)\frac{p^*+t-m}{p^*+t-s} = a+2\sqrt{3}\sigma\,\frac{p^*+t-m}{p^*+t-s}, \tag{48}$$

$$\begin{aligned} E(A^*) &= \int_a^{Q^*} xf(x)\,\mathrm{d}x + \int_{Q^*}^b Qf(x)\,\mathrm{d}x \\ &= K-\alpha p^* - \sqrt{3}\sigma + 2\sqrt{3}\sigma\,\frac{p^*+t-m}{p^*+t-s} - \sqrt{3}\sigma\left(\frac{p^*+t-m}{p^*+t-s}\right)^2. \end{aligned} \tag{49}$$

From the first order condition we also have

$$E(A^*) = \alpha(p^*-m), \tag{50}$$

$$w^* = \frac{1}{2Q^*}\left\{(p^*+t-s)\left[Q^*-\sqrt{3}\sigma\left(\frac{p^*+t-m}{p^*+t-s}\right)^2\right] + (s+m)Q^* - t(K-\alpha p^*)\right\}. \tag{51}$$

By combining (49) and (50),

$$K-2\alpha p^*+\alpha m = \sqrt{3}\sigma\left(\frac{p^*+t-m}{p^*+t-s}-1\right)^2. \tag{52}$$

Let $y = \frac{p^* + t - m}{p^* + t - s} 1$, substitute it into (48), (51) and (52), we get

$$p^* = s - t + (s - m)/y, \quad Q^* = K - \alpha p^* - \sqrt{3}\sigma + 2\sqrt{3}\sigma(y + 1) \quad \text{and}$$

$$w^* = \frac{1}{2Q^*}[(p^* + t - s)(Q^* - \sqrt{3}\sigma(y + 1)^2) + (s + m)Q^* - t(K - \alpha p^*)],$$

where $y^3 + Uy + V = 0$, where $U = -\frac{K + \alpha m + 2\alpha(t - s)}{\sqrt{3}\sigma}$, $V = \frac{2\alpha(s - m)}{\sqrt{3}\sigma}$. □

Proof of Theorem 2. Combining Lemma 1 and solving the second order condition of the cooperative game model (7), we have

$$\frac{(p^* + t - m)(p^* + t + m - 2s)}{(p^* + t - s)^2} + \frac{(p^* + t - s)\alpha}{\sqrt{3}\sigma} > 3. \tag{53}$$

Substituting (30) into (53) gives eventually

$$y^2 - \alpha(s - m)/(\sqrt{3}\sigma y) < 0. \tag{54}$$

From the assumption of $p > w > m > s$, we have

$$y = (s - m)/(p^* + t - s) < 0, \tag{55}$$

so (54) can be rewritten as

$$\sqrt{3}\sigma y^3 + \alpha(m - s) > 0. \tag{56}$$

Let $u = -[K + \alpha m + 2\alpha(t - s)]/(3\sqrt{3})$, $v = \alpha(s - m)/\sqrt{3}$, (33) can be rewritten as

$$\sigma y^3 = -3uy - 2v. \tag{57}$$

Combining (56) and (57) gives

$$3e < y < 2e. \tag{58}$$

In (58), $e = \alpha(s - m)/[K + \alpha m + 2\alpha(t - s)] < 0$.

Eq. (58) indicates that: if solutions of (33) satisfy $y \in (3e, 2e)$, then the equilibrium solutions (w^*, Q^*, p^*) satisfies the second order condition.

According to the property of the solutions of cubic equation (33) has a unique solution

$$y = -2\sqrt{\frac{\alpha(s - m)}{3\sqrt{3}\sigma e}} \cos\left(60^\circ + \frac{\theta}{3}\right) \in (3e, 2e), \quad \text{where } \theta = \arctan\sqrt{\frac{\alpha(s - m)}{27\sqrt{3}\sigma e^3} - 1}.$$

Thus, the cooperative game has a unique equilibrium solution (w^*, Q^*, p^*).

Eq. (33) can be reformulated as

$$\sigma = \frac{K + \alpha m + 2\alpha(t - s)}{\sqrt{3}y^2} + \frac{2\alpha(m - s)}{\sqrt{3}y^3}. \tag{59}$$

Similarly, (25) can be rewritten as

$$\mu^* = K - \alpha p^* = K + \alpha(t - s) + \alpha(m - s)/y, \tag{60}$$

$$c_D = \frac{\sigma}{\mu^*} = \frac{[K + \alpha m + 2\alpha(t - s)]y + 2\alpha(m - s)}{\sqrt{3}y^2[(K + \alpha(t - s))y + \alpha(m - s)]}. \tag{61}$$

From the monotone property of (61), we obtain

$$0 \leqslant c_D < c_D^*, \tag{62}$$

where

$$c_D^* = \frac{(17K + \alpha(16t - m - 16s))\sqrt{(-K + \alpha m)(-17K + \alpha(-16t + m + 16s))}}{16\sqrt{3}\alpha^2(m - s)^2} - \frac{[-71K^2 + 2\alpha K(-52t + 19m + 52s) + \alpha^2(-32t^2 + 40tm + m^2 + 64ts - 40ms - 32s^2)]}{16\sqrt{3}\alpha^2(m - s)^2}.$$

Also because $D \sim U[a, b]$, in general, $a = K - \alpha p - \sqrt{3}\sigma > 0$, so $c_D < 1/\sqrt{3}$. Combining this with (62), we obtain $0 \leqslant c_D < c_D^{\max}$, where $c_D^{\max} = \min\{1/\sqrt{3}, c_D^*\}$. □

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