PERGAMON

Automatica 35 (1999) 1417–1426

www.elsevier.com/locate/automatica

Brief Paper

Robust decentralized adaptive control for non-minimum phase systems with unknown and/or time varying delay☆

M. Makoudi*, L. Radouane

LESSI Faculté des Sciences, Département de Physique, B.P. 1796, Atlas, 30000, Morocco

Received 28 July 1997; revised 15 October 1998; received in final form 8 February 1999

Abstract

This paper presents a decentralized model reference adaptive control (DMRAC) for interconnected subsystems with unknown and/or time-varying time delay. The decentralization approach is based on the interconnection output estimation using the polynomial series which offers a general solution for interconnected subsystems. The parameter estimation scheme is a combined adaptive data filtering with a recursive least-squares algorithm with parameter projection and signal normalization. A "good data" model is defined by an adaptive filtering of the input and output signals. The obtained model permits to deal with non-minimum phased subsystems with unknown or time-varying dead time and at the same time to relax the hypothesis of weak interconnections for decentralized control. The performance of the studied scheme is illustrated by numerical examples. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Large-scale systems; Decentralized adaptive control; Non-minimum phase systems; Time-varying delay; Good data model; Interconnection prediction

1. Introduction

In the control of large-scale systems, one usually faces poor knowledge on the plant parameters and interconnections between subsystems. Thus the adaptive control technique in this case is an appropriate strategy to be employed. Moreover, if some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from the subsystems. Also, the design and implementation of the centralized controller may be complicated. An effective way to handle this difficulty is to apply decentralized control strategies, whereby each subsystem is controlled independently on the basis of its own performance criterion and locally available information. The majority of the results on decentralized control of large-scale systems described in input/output form refer to systems that consist of weakly coupled subsystems (Wen and Hill, 1992; Datta and Ioannou, 1991; Hejda et al., 1990; Ioannou, 1986; Ossman, 1989; Praly and Trulsson, 1986; Reed and Ioannou, 1988). The subsystems may be considered in isolation, and the local controllers received for the isolated subsystems may be applied as decentralized controllers to the overall system. However, the information which may be extracted from the interconnections are generally ignored and considered as perturbations (Ossman, 1989; Praly and Trulsson, 1986). No attempt has been made to estimate them in order to improve the control performance and robustness.

In this paper, we present a decentralized model reference adaptive control (DMRAC) for interconnected subsystems. The main idea is to predict the interconnection outputs acting on each subsystem to relax the hypothesis of weak interconnections. These predictions are used for the synthesis of the local control. The prediction method is based on expressing the interconnection outputs as a linear combination of a set of orthogonal known functions of a basis. Note that the polynomial approach has received much attention in mathematical modelling, identification and control literature (Heuberger et al., 1995; Lee and Tsay, 1986; Niedzwecki, 1988; Greblicki, 1994; James, 1994; Wei, 1990; Zervos et al., 1985, 1988; Zervos and Dumont, 1988). The parameter estimation

* Corresponding author. Tel.: (212)5-642389; fax: (212)5-642500; e-mail: makoudi@fesnet.net.ma.

☆ This paeper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor G. Dumont under the direction of Editor C.C. Hang.

PII: S0005-1098(99)00052-7

scheme is a combined adaptive data filtering with a recursive least-squares algorithm with parameter projection and signal normalization. The problem of minimum phase is handled by adjusting the data filtering parameter at each instant. Then all the subsystem zeros are relocated inside the unit circle. It is demonstrated that the input/output data filtering permits also to solve the difficult problem of model reference adaptive control in the presence of unknown time-delay variations, where only the upper limit on the time delay must be known. Then, relaxed constraints on the interconnections are necessary to ensure the global system boundedness. The scheme robustness with respect to unmodelled dynamics is also simultaneously improved. The paper is organized as follows: In Section 2, we present the system description. A robust DMRAC for non-minimum phase subsystems with varying time delay is presented in Section 3. In Section 4, the robustness of the DMRAC is established without a priori information about the exact invariant time delay. Numerical examples are finally given for illustration.

2. Problem statement

The system to be considered consists of N interconnected single-input–single-output (SISO) subsystems described in input/output form by

$$A_i(q^{-1})Y_i(t) = B_i(q^{-1})U_i(t-di) + \sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij}(q^{-1})Y_j(t-1) + \xi_i(t), \quad (1)$$

where $Y_i(t)$, $U_i(t)$ are the output and input, respectively, of the subsystem i. $A_i(q^{-1})$, $B_i(q^{-1})$ and $C_{ij}(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}.

$$A_i(q^{-1}) = 1 + a_{i(1)}q^{-1} + \cdots + a_{i(ni)}q^{-ni}, \quad (2)$$

$$B_i(q^{-1}) = b_{i(0)} + \cdots + b_{i(mi)}q^{-mi}, \quad b_{i(0)} \neq 0. \quad (3)$$

The polynomials $C_{ij}(q^{-1})$ are the interconnection inputs of $Y_i(t)$ from the other subsystems $Y_j(t)$ with

$$C_{ij}(q^{-1}) = c_{ij(0)} + \cdots + c_{ij(rij)}q^{-rij}. \quad (4)$$

The time-delay index di ($di \geq 1$), is unknown and/or time varying. $\xi_i(t)$ represents the unmodelled response of subsystem i.

Subsystem (1) can be expressed as

$$Y_i(t) = \Phi_i^*(t-1)^{\mathrm{T}}\theta_i^* + V_i(t), \quad (5)$$

where

$$V_i(t) = \sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij}(q^{-1})Y_j(t-1) + \xi_i(t).$$

$\Phi_i^*(t-1)^{\mathrm{T}}$ is a regression vector and θ_i^* denotes a vector containing unknown parameters of the nominal subsystem model, i.e.

$$\Phi_i^*(t-1)^{\mathrm{T}} = [Y_i(t-1)\ldots Y_i(t-ni) \quad U_i(t-di)\ldots U_i(t-di-mi)],$$

$$\theta_i^{*\mathrm{T}} = [-a_{i(1)}\cdots - a_{i(ni)} \quad b_{i(0)}\cdots b_{i(mi)}].$$

In this paper, the objective is to derive a decentralized control law to stabilize the global system with no information exchange among the subsystems. It is assumed that for subsystem i, the output $V_i(t)$ is not available. Thus, we propose that each subsystem predicts its interconnection outputs in real time. The prediction method is based on expressing the interconnection outputs as a linear combination of a set of known functions of a basis.

2.1. Functional series modelling

Let $T = \{1, 2, \ldots, \alpha\}$ and let $S(t) = \{s_1(t), \ldots, s_m(t)\}^{\mathrm{T}}$ be the set of linearly bounded independent sequences (functions of discrete-time t) on T. The elements of S will be further referred to basis functions. Possible sets of functions that could be used include powers of time $s_j(t) = t^{j-1}$, $j = 1, \ldots, m$ (which will be referred to as the Legendre basis) or cosine functions

$$S(t) = \begin{cases} \cos \pi(j-1)t & \text{if } j \text{ is odd} \\ \sin \pi(j-1)t & \text{if } j \text{ is even.} \end{cases}$$

(which is called the Fourier basis).

Now, we will use the functional modelling of the interconnections. We assume that for subsystem i, the unknown output $V_i(t)$ may be expanded into a series as

$$V_i(t) = M_i^{\mathrm{T}}(t)S(t) + W_i(t), \quad (6)$$

where $S(t) = [s_1(t), \ldots, s_m(t)]^{\mathrm{T}}$, $s_i(t)$ are known functions of a basis, $M_i(t) \in R^m$.

$W_i(t)$ is the residual between the output $V_i(t)$ and the series which may be unbounded. $M_i(t)$ is a time-varying vector of unknown elements to be estimated. The vector $M_i(t)$ is taken to be time varying, to represent more precisely the interaction term $V_i(t)$. This modelization is more adequate than $V_i(t) = M_i^{\mathrm{T}}S(t) + W_i(t)$ where M_i is a constant vector (see the Appendix).

By substitution of Eq. (6) into (1), we obtain

$$A_i(q^{-1})Y_i(t) = B_i(q^{-1})U_i(t-di) + M_i^{\mathrm{T}}(t)S(t) + W_i(t). \quad (7)$$

Further assumptions concerning the plant are made:

A1: An upper bound ni^* on the orders ni, mi and rij, $i, j = 1, 2, \ldots, N$ is known.

A2: There exists a known scalar $\rho_i \in R^+$ such that $\|\theta_i^*\| \leq \rho_i$.

A3: There exists a finite positive scalar μ_i such that: $|W_i(t)| \le \mu_i Z_i(t)$

$$Z_i(t) = \sigma_i Z_i(t-1) + \max[|Y_i(t-1)| + |U_i(t-1)| + |S(t)|, Z_{i0}],$$
$$0 < \sigma_i < 1,\ Z_{i0} > 0,\ Z_i(0) > 0.$$

In this paper, we plan to design a decentralized adaptive control which should stabilize the global system and cause $Y_i(t)$ to track a bounded reference model $Y_{im}(t)$. A major drawback with model reference adaptive control methods is that they require a priori knowledge of the dead time of the process transfer function which must be minimum phased. In many industrial processes, the time-delay di is either unknown accurately or even time varying.

3. DMRAC in presence of varying time delay

If the time-delay di is time-varying i.e. $dimin \le di(t) \le dimax$ where $dimin \ge 1$ and only $dimax$ is known, the normal approach in literature (Dumont, 1982; Vodel and Edgar, 1980) is to model subsystem (7) by

$$A_i(q^{-1})Y_i(t) = B_i'(q^{-1})U_i(t) + M_i^{\mathrm{T}}(t)S(t) + W_i(t) \tag{8}$$

with

$$B_i'(q^{-1}) = b_{i(1)}' q^{-1} + \cdots + b_{i(ni^* + dimax)}' q^{-ni^* - dimax}. \tag{9}$$

Subsystem (8) can be expressed as

$$Y_i(t) = \Phi_i^{\mathrm{T}}(t-1)\theta_i(t) + W_i(t), \tag{10}$$

where

$$\Phi_i^{\mathrm{T}}(t-1) = (Y_i(t-1) \ldots Y_i(t-ni^*)$$
$$U_i(t-1) \ldots U_i(t - dimax - ni^*) S^{\mathrm{T}}(t)),$$
$$\theta_i^{\mathrm{T}}(t) = (-a_{i(1)} \cdots - a_{i(ni^*)} \quad b_{i(1)}' \cdots b_{i(ni^* + dimax)}' \quad M_i^{\mathrm{T}}(t)). \tag{11}$$

Compared to model (7), this corresponds to an extension of the B-polynomial with ($dimax$-1) extra parameters.

3.1. Adaptive "good data" model

The key issue to get a robust parameter estimation may be viewed as a "good data" model and a robust parameter adaptation algorithm with respect to bounded disturbances and time-varying parameters. Let the Π_i linear operator be defined as (Makoudi and Radouane, 1995, 1997)

$$\Pi_i Y_i(t-k) = \alpha_i^k(t) Y_i(t-k), \tag{12}$$

$$\Pi_i U_i(t-k) = \alpha_i^k(t) U_i(t-k) \tag{13}$$

with $0 < \alpha_i(t) < 1\ \forall t$, where $\alpha_i(t)$ will be adjusted; this corresponds to an adaptive exponential filtering.

Given two asymptotically stable polynomials $O_i(q^{-1})$ and $P_i(q^{-1})$, and a Π_i operator, then operating on plant (8) by the filter $\Pi_i(O_i/P_i)$ leads to

$$\Pi_i A_i(q^{-1}) Y_{if}(t) = \Pi_i B_i'(q^{-1}) U_{if}(t) + \Pi_i M_i^{\mathrm{T}}(t) S_{\mathrm{f}}(t) + \Pi_i W_{if}(t) \tag{14}$$

with

$$O_i(q^{-1})(\,.\,)_i(t) = P_i(q^{-1})(\,.\,)_{if}(t).$$

The plant filtered model (14) may be expressed in regression form as follows:

$$Y_{if}(t) = \Phi_{if}^{\mathrm{T}}(t-1, \alpha_i)\theta_i(t) + W_i'(t) \tag{15}$$

$$\Phi_{if}^{\mathrm{T}}(t-1, \alpha_i) = (\alpha_i(t) Y_{if}(t-1) \ldots \alpha_i(t)^{ni^*} Y_{if}(t-ni^*) \alpha_i(t) U_{if}(t-1) \ldots \alpha_i(t)^{ni^* + dimax} U_{if}(t - ni^* - dimax) S_{\mathrm{f}}^{\mathrm{T}}(t)),$$

$$W_i'(t) = \Pi_i W_{if}(t).$$

Remarks. (1) The plant filtered model (14) hence involves the main design features. The input–output signals are filtered by $\Pi_i(O_i/P_i)$. The filter (O_i/P_i), which should be low-pass, is used to reduce the high-frequency modes of the unmodelled dynamics. The new "good data" model involves in spite of the low-pass filtering, an exponential adaptive filtering (Π_i). The Π_i operator allows to remove the minimum phase assumption while improving the robustness of the DMRAC (Lemma 3). This is equivalent to generating a minimum phase estimated model for each subsystem. Note that when $\alpha_i(t) = 1$ the proposed filter reduces to the classical one O_i/P_i considered in Giri et al. (1991) and Sripada and Fisher (1987).

(2) Note that in the previous works dealing with small interconnections (see, for example, Wen and Hill, 1992; Datta and Ioannou, 1991; Hejda et al., 1990; Praly and Trulsson, 1986), the classical assumption made for Eq. (5) is $|V_i(t)| \le \mu_i Z_i(t)$ where $Z_i(t)$ depends only on the locally available information. μ_i is required to be sufficiently small for stability purpose. In our work, only the residual error $W_i(t)$ (Eq. (8)) is constrained to satisfy assumption A3. This is due to the introduction of the term ($S(t)$) in the observation vector and of the vector $M_i(t)$ in the parameter vector (Eq. (11)). The interconnections are thus taken into account by each subsystem. Then the normalizing signal $Z_i(t)$ depends on local subsystem signals and implicitly on the interconnection signals also. Therefore, this new assumption makes the present work different from the previous ones on the same subject. Assumption A3 is no more restrictive.

Lemma 1. *Consider subsystem* (7) *subject to* A3, *then for subsystem model* (14), *there exists a scalar* $\mu_i' > 0$ *such that*

$$|W_i'(t)| \le |\mu_i' Z_{if}(t),$$

$$Z_{if}(t) = \sigma_i Z_{if}(t-1) + \max(|Y_{if}(t-1)| + |U_{if}(t-1)| + |S_{\mathrm{f}}(t)|, Z_{i0}), \quad 0 < \sigma_i < 1,\ Z_{i0} > 0,\ Z_{if}(0) > 0.$$

Thus, the filter $\Pi_i(O_i/P_i)$ permits to reduce the effects of the unmodeled dynamics.

Proof. Consider the filtered subsystem model defined by:

$$\Pi_i A_i(q^{-1})Y_{if}(t) = \Pi_i B_i'(q^{-1})U_{if}(t) + \Pi_i M_i^{T}(t)S_f(t) + W_i'(t),$$

where

$$W_i'(t) = \Pi_i W_{if}(t) = V_i'(t) - M_i^{T}(t)S_f(t),$$

$$V_i'(t) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \Pi_i(c_{ij0} + \cdots + c_{ijrij})Y_{jf}(t-1) + \Pi_i \xi_{if}(t).$$

Then

$$V_i'(t) = \sum_{\substack{j=1 \\ j\neq i}}^{N} (c_{ij0}\alpha_i(t) + \cdots + c_{ijrij}\alpha_i^{rij}(t)q^{-rij})Y_{jf}(t-1) + \xi_i'(t).$$

The unmodelled dynamics $\{\xi_i(t)\}$ may contain non-linearities and bounded disturbances. We will consider, without loss of generality, the following class:

$$\xi_i(t) = \Delta A_i(t)Y_i(t-1) + \Delta B_i(t)U_i(t-1) + F_i(t) + \eta_i(Y_i(t-1) + U_i(t-1))^2,$$

where

$$\|\Delta A_i(t)\| \le \kappa_{i1}, \quad \|\Delta B_i(t)\| \le \kappa_{i2}, \quad \|F_i(t)\| \le \kappa_{i3},$$

where

$$\xi_{if}(t) = \Delta A_i(t)Y_{if}(t-1) + \Delta B_i(t)U_{if}(t-1) + F_{if}(t) + \eta_i(Y_{if}(t-1) + U_{if}(t-1))^2$$

and

$$\xi_i'(t) = \Delta A_i(t)\alpha_i(t)Y_{if}(t-1) + \Delta B_i(t)\alpha_i(t)U_{if}(t-1) + \alpha_i(t)F_{if}(t) + \eta_i(\alpha_i(t)Y_{if}(t-1) + \alpha_i(t)U_{if}(t-1))^2,$$

then

$$|\xi_i'(t)| < \alpha_i(t)|\xi_{if}(t)|$$

and

$$|V_i'(t)| < \alpha_i(t)\left|\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (c_{ij0} + \cdots + c_{ijrij}q^{-rij})Y_{jf}(t-1) + \xi_{if}(t)\right)\right| < \alpha_i(t)|V_{if}(t)|.$$

From assumption A4, we have $|W_i(t)| \le \mu_i Z_i(t)$ then $|W_{if}(t)| \le \mu_i Z_{if}(t)$ and $|W_i'(t)| \le \alpha_i(t)\mu_i Z_{if}(t) = \mu_i' Z_{if}(t)$. As $0 < \alpha_i(t) < 1$, the bound μ_i' decreases, so large unmodelled dynamics may be tolerated and the classical weak interconnection assumption is relaxed.

3.2. Robust parameter adaptation algorithm

Let $\hat\theta_i(t)$ denote the estimate of $\theta_i(t)$ at time t using the least-squares parameter estimation with parameter projection, signal normalization and data filtering (Wen and David, 1992).

$$\hat\theta_i(t) = \Im\left\{\hat\theta_i(t-1) + \frac{\Phi_{if}(t-1,\alpha_i)e_{if}(t)}{Z_{if}^2(t) + \Phi_{if}^{T}(t-1,\alpha_i)\Phi_{if}(t-1,\alpha_i)}\right\}, \tag{16}$$

where $\Im$ represents the projection operator necessary to ensure $\|\hat\theta_i(t)\| \le \rho_i$ for all t if $\|\hat\theta_i(0)\| \le \rho_i$. $e_{if}(t)$ is the prediction error defined as

$$e_{if}(t) = Y_{if}(t) - \hat\theta_i^{T}(t-1)\Phi_{if}(t-1,\alpha_i). \tag{17}$$

Some useful properties of estimator (16), (17) are now given in Lemma 2.

Lemma 2. *Consider the system model given in* (8), *subject to assumptions* A1–A3, *then the algorithm* (16)–(17) *has the following properties*:

(a) $\|\hat\theta_i(t)\| \le \rho_i$.
(b) *There exists a positive constant* ρ_{ie} *such that for all* $(l, v) \in N$ *one has*

$$\sum_{t=l+1}^{l+v} \frac{|e_{if}(t)|}{Z_{if}(t)} \le \rho_{ie} + v\mu_i.$$

(c) *There exists a positive constant* ρ_{io} *such that for all* $(l, v) \in N$ *one has*

$$\sum_{t=l+1}^{l+v} \|\hat\theta_i(t) - \hat\theta_i(t-1)\| \le \rho_{io} + v\mu_i.$$

The proof of Lemma 2 may be carried out along the same lines as in (Giri et al., 1991) and is then omitted.

Given $\hat\theta_i(t)$, we form $\hat A_{i\alpha}(t,q^{-1})$, $\underline{\hat B}'_{i\alpha}(t,q^{-1})$ from the coefficients as

$$\hat A_{i\alpha}(t,q^{-1}) = 1 + \hat a_{i(1)}(t)\alpha_i(t)q^{-1} + \cdots + \hat a_{i(ni^*)}(t)\alpha_i(t)^{ni^*}q^{-ni^*}, \tag{18}$$

$$\underline{\hat B}'_{i\alpha}(t,q^{-1}) = \alpha_i(t)\hat b'_{i(1)}(t) + \cdots + \alpha_i^{ni^*+dimax}(t)\,\hat b'_{i(ni^*+dimax)}(t)q^{-ni^*-dimax+1}. \tag{19}$$

Lemma 3. *Consider the system* (8), *subject to assumptions* A1 *and* A2, *if* $|\alpha_i(t)| < |\hat b'_{i(1)}(t)|/(\rho_i\sqrt{ni^* + dimax - 1})$ $\forall t$, *then* $\underline{\hat B}'_{i\alpha}(t,q^{-1})$ *is a stable polynomial.*

Proof. $\underline{\hat B}'_{i\alpha}(t,q^{-1})$ is a stable polynomial if

$$\sum_{k=2}^{ni*+dimax} \frac{\hat b'^2_{i(k)}(t)}{\hat b'^2_{i(1)}(t)}\alpha_i^{2(k-1)}(t) \le \frac{1}{ni^* + dimax - 1}$$

(see Zinober, 1983), where $ni^* + dimax$ is the order of $\underline{\hat{B}}'_{i\alpha}(t, q^{-1})$. As $|\alpha_i(t)| \le 1$, we get

$$\sum_{k=2}^{ni^*+dimax} \frac{\hat{b}'^2_{i(k)}(t)}{\hat{b}'^2_{i(1)}(t)} \alpha_i^{2k-2}(t) \le \alpha_i^2(t) \sum_{k=2}^{ni^*+dimax} \frac{\hat{b}'^2_{i(k)}(t)}{\hat{b}'^2_{i(t)}(t)} \le \frac{1}{ni^* + dimax - 1}$$

and

$$\sum_{k=1}^{ni^*+dimax} \hat{b}^2_{i(k)}(t) < \|\hat{\theta}_i(t)\|^2 < \rho_i^2$$

then, the condition $|\alpha_i(t)| < |\hat{b}'_{i(1)}(t)|/(\rho_i\sqrt{ni^* + dimax - 1})$ assures the stability of $\underline{\hat{B}}'_{i\alpha}(t, q^{-1})$.

3.3. Decentralized adaptive control

The proposed feedback control signal is generated from

$$\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1})U_{if}(t) + \hat{G}_i(t, q^{-1})Y_{if}(t) + \hat{M}_i^{\mathrm{T}}(t)S_f(t+1) = T_i(q^{-1})Y_{imf}(t+1). \tag{20}$$

$Y_{im}(t)$ is the given bounded output of the reference model satisfying

$$O_i(q^{-1})Y_{im}(t) = P_i(q^{-1})Y_{imf}(t). \tag{21}$$

$T_i(q^{-1})$ is an arbitrary stable monic polynomial of degree nti, and

$$\hat{G}_i(t, q^{-1}) = \hat{g}_{i(0)}(t) + \cdots + \hat{g}_{i(ngi)}(t)q^{-ngi}, \tag{22}$$

where $nti < ni^* + 1$ and $ngi < ni^*$.

The term $\hat{M}_i^{\mathrm{T}}(t)S_f(t+1)$ is introduced because of the existence of the interconnections and is then used for their effect reduction.

The estimation error of the subsystem i is given by

$$\begin{aligned} e_{if}(t) &= Y_{if}(t) - \hat{\theta}_i^{\mathrm{T}}(t-1)\Phi_{if}(t-1, \alpha_i) \\ &= \hat{A}_{i\alpha}(t-1, q^{-1})Y_{if}(t) - \underline{\hat{B}}'_{i\alpha}(t-1, q^{-1})U_{if}(t-1) \\ &\quad - \hat{M}_i^{\mathrm{T}}(t-1)S_f(t). \end{aligned} \tag{23}$$

Adding and subtracting $q^{-1}.\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1})U_{if}(t)$ in Eq. (23) gives

$$\begin{aligned} \hat{A}_{i\alpha}(t-1, q^{-1})Y_{if}(t) &= q^{-1}\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1})U_{if}(t) + \hat{M}_i^{\mathrm{T}}(t)S_f(t) \\ &\quad + e_{if}(t) + [\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1}).q^{-1} \\ &\quad - q^{-1}.\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1})]U_{if}(t) \end{aligned} \tag{24}$$

and in view of Eq. (20)

$$\begin{aligned} \hat{A}_{i\alpha}(t-1, q^{-1})Y_{if}(t) &= q^{-1}(T_i(q^{-1})Y_{imf}(t+1) \\ &\quad - \hat{M}_i^{\mathrm{T}}(t)S_f(t+1) - \hat{G}_i(t, q^{-1})Y_{if}(t)) \\ &\quad + e_{if}(t) + \hat{M}_i^{\mathrm{T}}(t-1)S_f(t) \\ &\quad + [\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1}).q^{-1} \\ &\quad - q^{-1}\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1}]U_{if}(t). \end{aligned} \tag{25}$$

$\hat{G}_i(t, q^{-1})$ is determined by solving the following equality:

$$\hat{A}_{i\alpha}(t-1, q^{-1}) + q^{-1}\hat{G}_i(t, q^{-1}) = T_i(q^{-1}). \tag{26}$$

The closed-loop control system equation is then

$$\begin{aligned} T_i(q^{-1})[Y_{if}(t) - Y_{imf}(t)] &= e_{if}(t) + (\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1}).q^{-1} \\ &\quad - q^{-1}.\underline{\hat{B}}'_{i\alpha}(t-1, q^{-1}))U_{if}(t). \end{aligned} \tag{27}$$

The indirect adaptive control algorithm above is globally convergent, as shown below.

Theorem 1. *Consider the decentralized adaptive control system consisting of the plant* (8), *subject to assumptions* A1–A3 *in closed-loop with the adaptive control law* (20). *Then, the resulting closed-loop system is globally stable in the sense that*:

(i) *$U_i(t)$, $Y_i(t)$ are bounded for all time,*
(ii) *if μ_i is equal to zero, then $\lim(Y_i(t) - Y_{im}(t)) = 0$ as $t \to \infty$*

Proof. We define $\hat{A}.\hat{B}$ and $\hat{A}\hat{B}$

$$\hat{A}.\hat{B} = \sum_i \sum_j \hat{a}_i(t)\hat{b}_j(t-i)q^{-i-j} \ne \hat{B}.\hat{A}, \tag{28}$$

$$\hat{A}\hat{B} = \sum_i \sum_j \hat{a}_i(t)\hat{b}_j(t)q^{-i-j} = \hat{B}\hat{A}. \tag{29}$$

We also define

$$\bar{B} = \hat{B}(t-1, q^{-1}). \tag{30}$$

From (17), we have

$$e_{if}(t+1) = \hat{A}_{i\alpha}Y_{if}(t+1) - \underline{\hat{B}}'_{i\alpha}U_{if}(t) - \hat{M}_i^{\mathrm{T}}(t)S_f(t+1), \tag{31}$$

$$\begin{aligned} e_{if}(t+1) &= TY_{if}(t+1) - \hat{G}_iY_{if}(t) - \underline{\bar{B}}'_{i\alpha}U_{if}(t) \\ &\quad - \hat{M}_i^{\mathrm{T}}(t)S_f(t+1) + [\hat{A}_{i\alpha} - \bar{A}_{i\alpha}]Y_{if}(t+1) \\ &\quad - [\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}]U_{if}(t) \end{aligned} \tag{32}$$

or

$$\begin{aligned} &TY_{if}(t+1) + [\hat{A}_{i\alpha} - \bar{A}_{i\alpha}]Y_{if}(t+1) - [\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}]U_{if}(t) \\ &\quad = e_{if}(t+1) + T_iY_{imf}(t+1). \end{aligned} \tag{33}$$

Operating on (33) by $\hat{A}_{i\alpha}$ gives

$$\begin{aligned} &T\hat{A}_{i\alpha}Y_{if}(t+1) + \hat{A}_{i\alpha}.[\hat{A}_{i\alpha} - \bar{A}_{i\alpha}]Y_{if}(t+1) \\ &\quad - \hat{A}_{i\alpha}.[\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}]U_{if}(t) = \hat{A}_{i\alpha}e_{if}(t+1) + T_i\hat{A}_{i\alpha}Y_{imf}(t+1). \end{aligned} \tag{34}$$

Using (31) in (34), then (33) and (34) can be summarized as

$$\begin{bmatrix} T_i + [\hat{A}_{i\alpha} - \bar{A}_{i\alpha}] & -[\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}] \\ \hat{A}_{i\alpha}.[\hat{A}_{i\alpha} - \bar{A}_{i\alpha}] + [\hat{A}_{i\alpha}.T_i - \bar{A}_{i\alpha}T_i] & T_i\underline{\hat{B}}'_{i\alpha} - \hat{A}_{i\alpha}.[\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}] - T_i[\underline{\hat{B}}'_{i\alpha} - \underline{\bar{B}}'_{i\alpha}] \end{bmatrix}$$

$$\begin{bmatrix} Y_{if}(t+1) \\ U_{if}(t) \end{bmatrix} = \begin{bmatrix} T_i \\ T_i\hat{A}_{i\alpha} \end{bmatrix} Y_{imf}(t+1) + \begin{bmatrix} 1 \\ \hat{A}_{i\alpha} - T_i \end{bmatrix} e_{if}(t+1) + \begin{bmatrix} 0 \\ -T_i\hat{M}_i^T \end{bmatrix} S_f(t+1). \quad (35)$$

Eq. (35) can be regarded as a linear time varying system having inputs $\{e_{if}(t+1)\}$ and $\{Y_{imf}(t+1)\}$ and outputs $\{U_{if}(t)\}$ and $\{Y_{if}(t)\}$. The terms in square brackets, for example, $[\hat{A}_{i\alpha} - \bar{A}_{i\alpha}]$, and so on, arise due to the time-varying nature of the parameter estimates and $\alpha_i(t)$. Since $Y_{imf}(t)$, $S_f(t)$ are bounded and following part (iii) of Lemma 2, we can conclude that model (35) is asymptotically time invariant and stable provided that T_i and $\underline{\hat{B}}'_{i\alpha}$ are both stable. Thus, from (35), $\{U_{if}(t)\}$and $\{Y_{if}(t)\}$ are asymptotically bounded by $\{e_{if}(t)\}$. If μ_i is equal to zero, $\{e_{if}(t)\}$ converges to zero and $Y_{if}(t)$ converges to $Y_{imf}(t)$. Then $Y_i(t)$ converges to $Y_{im}(t)$. This establishes the theorem.

4. DMRAC in presence of unknown time delay

In this section, we consider the non-minimum phase subsystem described by

$$A_i(q^{-1})Y_i(t) = B_i(q^{-1})U_i(t-di) + M_i^{\mathrm{T}}(t)S(t) + W_i(t), \quad (36)$$

where the time-delay di is constant and unknown.

Eq. (36) can expressed as

$$A_i(q^{-1})Y_i(t) = b_{i(1)}U_i(t-1) + \cdots + b_{i(di)}U_i(t-di) + \cdots + b_{i(ni^*)}U_i(t-ni^*) + M_i^{\mathrm{T}}(t)S(t) + W_i(t)$$

or

$$A_i(q^{-1})Y_i(t) = B'_i(q^{-1})U_i(t) + M_i^{\mathrm{T}}(t)S(t) + W_i(t), \quad (37)$$

where

$$B'_i(q^{-1}) = b_{i(1)}q^{-1} + \cdots + b_{i(ni^*)}q^{-ni^*}. \quad (38)$$

According to De Keyser (1986), the following condition on the coefficients $b_{i(j)}$ holds:

$$|b_{i(j)}| < 0.2|b_{i(di)}| \quad \text{for } j = 0, \ldots, di-1, \quad (39)$$

Then, operating on the plant model (37) by the filter $\Pi_i(O_i/P_i)$ leads to

$$\Pi_i A_i(q^{-1})Y_{if}(t) = \Pi_i B'_i(q^{-1})U_{if}(t) + \Pi_i M_i^{\mathrm{T}}(t)S_f(t) + W'_i(t). \quad (40)$$

In regression form we have

$$Y_{if}(t) = \Phi_{if}^{\mathrm{T}}(t-1,\alpha_i)\theta_i(t) + W'_i(t),$$

$$\Phi_{if}^{\mathrm{T}}(t-1,\alpha_i) = (\alpha_i(t)Y_{if}(t-1)\cdots\alpha_i(t)^{ni^*}Y_{if}(t-ni^*)$$
$$\alpha_i(t)U_{if}(t-1)\cdots\alpha_i(t)^{ni^*}U_{if}(t-ni^*)S_f(t)).$$

The unknown process parameters $\theta_i(t)$ are estimated using the algorithms (16) and (17).

Given $\hat{\theta}_i(t)$,we form $\underline{\hat{B}}'_{i\alpha}(t,q^{-1})$ as

$$\underline{\hat{B}}'_{i\alpha}(t,q^{-1}) = \alpha_i(t)\hat{b}_{i(1)}(t) + \cdots + \alpha_i(t)^{ni^*}\hat{b}_{i(ni^*)}(t)q^{-ni^*}. \quad (41)$$

From Lemma 3, $\underline{\hat{B}}'_{i\alpha}(t,q^{-1})$ is a stable polynomial if $|\alpha_i(t)| < |\hat{b}_{i(1)}(t)|/\rho_i\sqrt{ni^*-1}$.

After filtering, given that $0 < \alpha_i(t) < 1$, condition (39) becomes

$$\alpha_i(t)\hat{b}_{i(1)}(t) > \alpha_i^2(t)\hat{b}_{i(2)}(t) > \cdots > \alpha_i^{ni^*}(t)\hat{b}_{i(ni^*)}(t).$$

Then the subsystem model (36) has the same behaviour as model (40) with time-delay $di = 1$.

The proposed feedback control signal is generated from

$$\underline{\hat{B}}'_{i\alpha}(t-1,q^{-1})U_{if}(t) + \hat{G}_i(t,q^{-1})Y_{if}(t) + \hat{M}_i^{\mathrm{T}}(t)S_f(t+1)$$
$$= T_i(q^{-1})Y_{imf}(t+1).$$

The indirect model reference adaptive control is globally convergent in the sense that $U_i(t)$, $Y_i(t)$ are bounded for all time and $Y_i(t)$ converges to $Y_{im}(t)$ (Theorem 1).

5. Numerical examples

Two examples that illustrate our results are given in this section. In the first example, we consider three interconnected non-minimum phase subsystems with time-varying delay $di(t) \in [dimin, dimax]$. The second example is used to demonstrate the performance of the DMRAC for non-minimum phase subsystems without a priori information about the constant time delay.

Example 1. To illustrate the performance and robustness of the proposed adaptive scheme, let us consider first the following interconnected subsystems with time-varying delay, to which we apply the adaptive decentralized control with the interconnection estimation.

$$A_1(q^{-1})Y_1(t) = B_1(q^{-1})U_1(t-d1(t)) + C_{12}(q^{-1})Y_2(t) + C_{13}(q^{-1})Y_3(t) + \xi_1(t),$$

$$A_2(q^{-1})Y_2(t) = B_2(q^{-1})U_2(t-d2(t)) + C_{21}(q^{-1})Y_1(t) + C_{23}(q^{-1})Y_3(t) + \xi_2(t),$$

$$A_3(q^{-1})Y_3(t) = B_3(q^{-1})U_3(t - d3(t)) + C_{31}(q^{-1})Y_1(t) + C_{32}(q^{-1})Y_2(t) + \xi_3(t)$$

with

$$A_1(q^{-1}) = 1 + 0.905q^{-1} - 0.3q^{-2} + 0.2q^{-3},$$

$$B_1(q^{-1}) = 1 + 1.5q^{-1}, C_{12}(q^{-1}) = -0.5q^{-1} - 0.7q^{-2} = C_{13}(q^{-1}), \quad 1 \le d1(t)4,$$

$$\xi_1(t) = 0.1Y_1(t-4) + 0.01\cos(2t)U_1(t-1) + 0.01\sin(2t)U_1(t-2)$$

$$A_2(q^{-1}) = 1 + 0.805q^{-1} - 0.1q^{-2} + 0.02q_2^{-3,B}(q^{-1}) = 1 + 1.5q^{-1}\ C_{21}(q^{-1}) = -0.5q^{-1} - 0.7q^{-2} = C_{23}(q^{-1}), \quad 2 \le d2(t)4,$$

$$\xi_2(t) = 0.01Y_2(t-4) + 0.01\cos(2t)U_2(t-1) + 0.01\sin(2t)U_2(t-2),$$

$$A_3(q^{-1}) = 1 + 0.995q^{-1} - 0.2q^{-2} + 0.1q^{-3},$$

$$B_3(q^{-1}) = 1 - 1.5q^{-1}\ C_{31}(q^{-1}) = -0.5q^{-1} + 0.7q^{-2} = C_{32}(q^{-1}), \quad 2 \le d3(t)5,$$

$$\xi_3(t) = 0.05Y_3(t-4) + 0.01\cos(2t)U_3(t-1) + 0.01\sin(2t)U_3(t-2).$$

Clearly, the subsystems are non-minimum phased. Suppose that we know $\rho_1 = 4$, $\rho_2 = 5$, $\rho_3 = 6$ and the reference signals $Y_{im}(t)$ are square waves with periods 300, 200 and 100, respectively. The data filtering parameters are chosen as $|\alpha_i(t)| = |\hat{b}'_{i(1)}(t)|/4\rho_i$ $i = 1, 2, 3$ and $S(t)$ is the Legendre series $S(t) = (1, t, t^2, t^3, \ldots)$. The low-pass filter (O_i/P_i) is such that

$$O_i(q^{-1}) = 1 + 0.1q^{-1} - 0.1q^{-2}, P_i(q^{-1}) = 1 - 0.2q^{-1} + 0.2q^{-2} \quad \text{for } i = 1, 2, 3.$$

Figs. 1(1), 1(3) and 1(5) show the system output. Figs. 1(2), 1(4) and 1(6) show the time-delay variations. The interconnections are estimated using the Legendre series truncated after three terms. We note that without input/output data filtering, and the interconnection compensation, the overall system is unstable (see Figs. 1(7)–1(9).

Now, we consider again the first example with added noise on the subsystem outputs. To each output, a Gaussia noise with mean = 3 and variance = 5 (Fig. 1(10)) is added. Under the same conditions of simulation as before, and for a low-pass filter $O_i(q^{-1}) = 1 + 0.01q^{-1} - 0.01q^{-2}$, $P_i(q^{-1}) = 1 - 0.2q^{-1} + 0.2q^{-2}$ for $i = 1, 2, 3$, the system behaviour is given by Figs. 1(11), 1(12) and 1(13). This illustrates the robustness of the proposed approach against noise and unmodelled dynamics.

Example 2. To show the performance of the DMRAC without a priori information about the exact time-delay *di*, we consider the same system as in Example 1 with unknown time-delay *di* ($di \ge 1$).

The system is described by

$$A_1(q^{-1})Y_1(t) = B_1(q^{-1})U_1(t - d1) + C_{12}(q^{-1})Y_2(t) + C_{13}(q^{-1})Y_3(t) + \xi_1(t),$$

$$A_2(q^{-1})Y_2(t) = B_2(q^{-1})U_2(t - d2) + C_{21}(q^{-1})Y_1(t) + C_{23}(q^{-1})Y_3(t) + \xi_2(t),$$

$$A_3(q^{-1})Y_3(t) = B_3(q^{-1})U_3(t - d3) + C_{31}(q^{-1})Y_1(t) + C_{32}(q^{-1})Y_2(t) + \xi_3(t),$$

where

$$B_1(q^{-1}) = b_{14} + b_{15}q^{-1},\ d1 = 4,\ B_2(q^{-1}) = b_{23} + b_{24}q^{-1},$$

$$d2 = 3,\ B_3(q^{-1}) = b_{32} + b_{33}q^{-1} \text{ and } d3 = 2.$$

Under the same conditions of simulation, we obtain the subsystem behaviour shown in Figs. 2(1)–2(3).

As illustrated by the previous examples, by applying an adaptive input–output data filtering, the DMRAC scheme can be constructed even for non-minimum phase subsystems with unknown and/or time-varying delay in the presence of unmodelled dynamics. Simulations show excellent performance and robustness of the DMRAC against unmodelled dynamics and time delay variations. In all cases, excellent tracking was achieved.

6. Conclusion

In this paper, we have proposed a decentralized model reference adaptive control for interconnected non minimum phase subsystems with time-varying delay. The approach is based on the interconnection output estimation using the polynomial series. The parameter estimation scheme is a combined data filtering with a recursive least-squares algorithm with projection and signal normalization. The problem of minimum phase of the subsystems is handled by an adaptive data filtering. It is shown that the adaptive input/output data filtering also permits to solve the difficult problem of model reference adaptive control in the presence of unknown time delay variations.

Appendix

General discrete orthogonal polynomials $s_i(t)$ satisfy the orthogonality property

$$\sum_{t=0}^{m-1} s_i(t)s_j(t) = \delta_{ij}, \quad i, j = 1, \ldots, m.$$

They satisfy the following recurrence relation:

$$s_{i+1}(t) = \varphi_i(t)s_i(t) + \omega_i s_{i-1}(t), \quad i = 1, \ldots, m,$$
$$t = 0, \ldots, m-1. \qquad (*)$$

$\varphi_i(t)$ *and* ω_i are the recurrence coefficients; their values depend on the particular discrete polynomials under consideration. We assume that for subsystem i, the unknown output $V_i(t)$ may be expanded into a series of

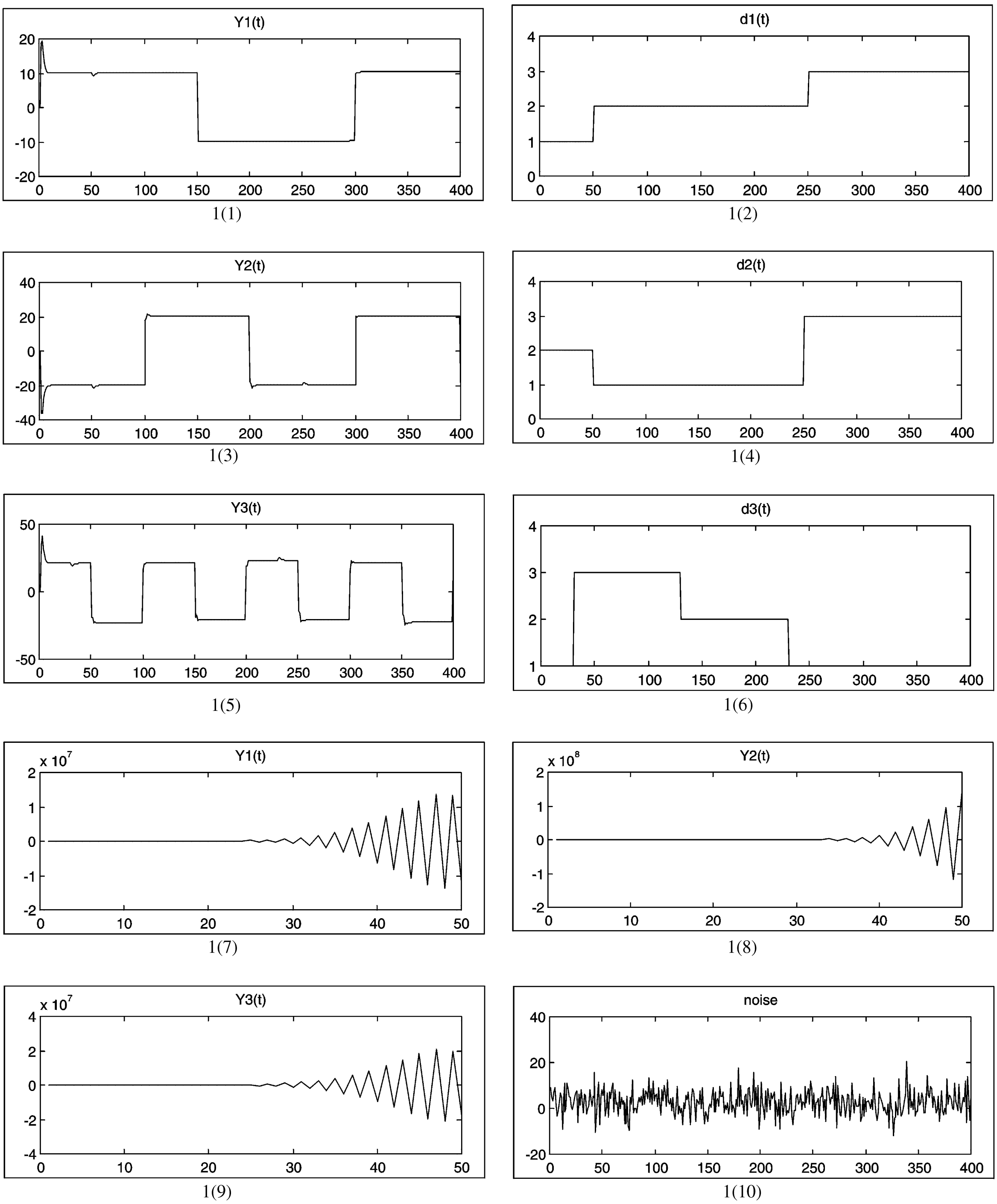
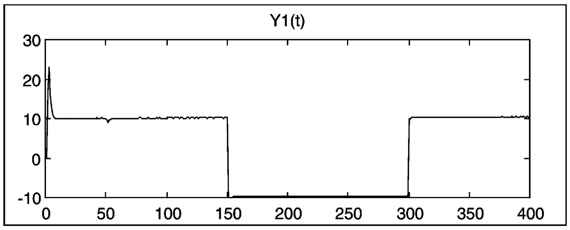
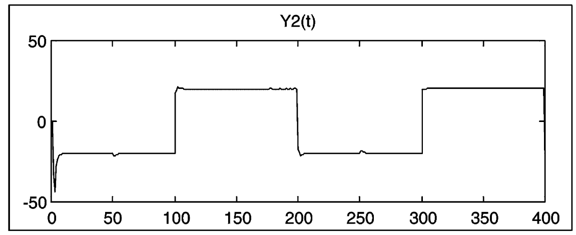


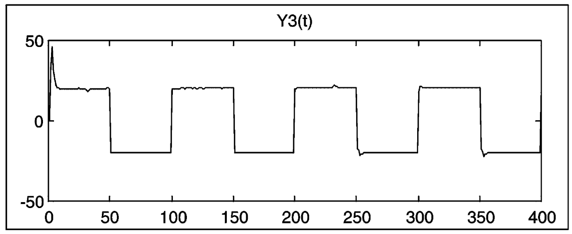
Fig. 1. (1) The first subsystem output for Example 1; (2) the first subsystem time-delay $d1(t)$ variation for Example 1; (3) the second subsystem output $Y_2(t)$ for example 1; (4) Tte second subsystem time-delay $d2(t)$ variation for Example 1; (5) the third subsystem output $Y_3(t)$ for Example 1; (6) the third subsystem time-delay $d3(t)$ variation for Example 1; (7) the first subsystem output $Y_1(t)$ for Example 1 without input/output data filtering, and the interconnection compensation; (8) the second subsystem output $Y_2(t)$ for Example 1 without input/output data filtering, and the interconnection compensation; (9) the third subsystem output $Y_3(t)$ for Example 1 without input/output data filtering, and the interconnection compensation; (10) the Gaussia noise with mean = 3 and variance = 5 which is added to each subsystem output for Example 1; (11) the first subsystem output $Y_1(t)$ for example1for which the noise (Fig. 1(10)) is added; (12) the second subsystem output $Y_2(t)$ for Example 1 for which the noise (Fig. 1(10)) is added; (13) the third subsystem output $Y_3(t)$ for Example 1 for which the noise (Fig. 1(10)) is added.



1(11)

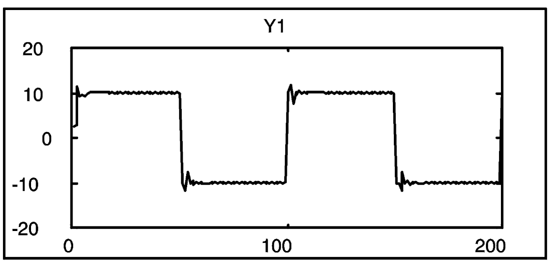


1(12)

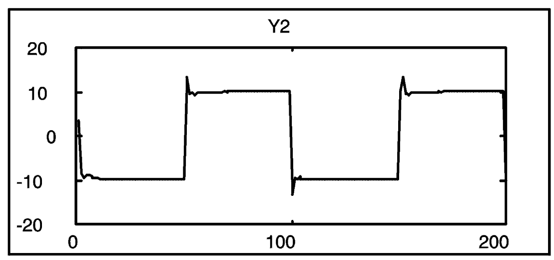


1(13)

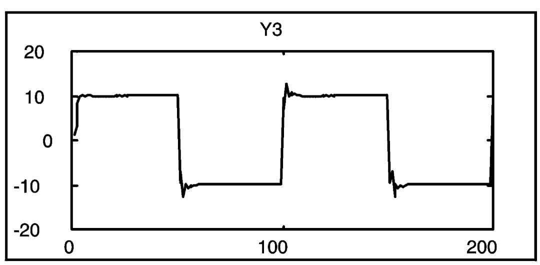
Fig. 1. Continued.



2(1)



2(2)



2(3)

Fig. 2. (1) The first subsystem output $Y_1(t)$ for Example 2 with unknown time delay; (2) the second subsystem output $Y_2(t)$ for Example 2 with unknown time delay; (3) the third subsystem output $Y_3(t)$ for Example 2 with unknown time delay.

order m as

$$V_i(t) = M_i^{\mathrm{T}} S(t) + \bar{W}_i(t) \quad \text{where } S(t) = [s_1(t), \ldots, s_m(t)]^{\mathrm{T}}.$$

Now, consider the following series with expansion order υ: $S'(t) = [s_1(t), \ldots, s_\upsilon(t)]^{\mathrm{T}}$, where $\upsilon > m$. then

$$V_i(t) = M_i'^{\mathrm{T}} S'(t) + W_i(t),$$

where $|W_i(t)| < |\bar{W}_i(t)|$ and the parameter vectors M_i and M_i' are assumed to be constant.

Using the recurrence equation (*), we obtain

$$s_{m+1}(t) = \varphi_m(t) s_m(t) + \omega_m s_{m-1}(t)$$

and

$$s_\upsilon(t) = \varphi_\upsilon(t) s_m(t) + \omega_\upsilon s_{m-1}(t).$$

Then the expansion of $V_i(t)$ into a series of order where M_i' is a constant vector of dimension υ, is equivalent to an expansion of order m where M_i is a time-varying vector of dimension $m < \upsilon$. This explains why a truncated series with time-varying parameters is more adequate to predict the interconnections with sufficient accuracy:

$$V_i(t) = M_i^{\mathrm{T}}(t) S(t) + W_i(t).$$

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Meryem Makoudi was born in Fez, Morocco on 11 October 1963. She graduated from the University sidi Mohamed Ben Abdellah, Faculty of Science, Fez, and received the "Thèse de 3ème cycle" degree in Automatic Control and Informatics in 1992. Since 1993, she has been a Professor of informatics at the faculty of legal, economical and social sciences, Fez. Her current research interests are in Large-scale systems, Decentralized control, Adaptive control, Nonlinear systems, Electrical networks, Irrigation canals and Industrial Applications.

Larbi Radouane was born in Sefrou, Morocco in 1953. He graduated from the University Mohamed V, Faculty of Science Rabat, and received the "Thèse de 3ème Cycle" and "Thèse d'Etat" degrees in 1980 and 1985, respectively. Since 1985, he has been a Professor of Automatic control and signal processing at the University sidi Mohamed ben Abdellah, Faculty of Science, Fez where he is presently Directeur de recherche and Head of the Laboratoire d'Electronique, Signaux Systèmes et d'Informatique (LESSI). His current research interests are in Large scale systems, Decentralized Control, Adaptive control, Signal and Image processing, and Industrial Applications.