

## Investigating the use of probability distribution functions in reliability-worth analysis of electric power systems

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### ABSTRACT

The use of probability distribution functions to describe reliability-worth input parameters is fairly new compared to using average values. Reliability-worth indices of power systems are frequently calculated as average values and convey little information about risk. In this paper beta probability distribution function was used to model time-dependent customer interruption costs as an input parameter to reliability-worth analyses of power systems. Time-sequential Monte-Carlo simulation technique that can handle time dependence of the input parameters was employed in the analysis. The results revealed that more information can be derived from the reliability-worth indices when probability distributions are used to describe the reliability-worth input and output parameters.

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### 1. Introduction

Power systems comprising generation, transmission and distribution, are subjected to many adverse events such as accidents, random component failures and weather conditions resulting in power interruptions. These kinds of events are beyond the control of a utility, but they can be taken into account when deciding the level of supply reliability at which the system should operate. In order to relate investment costs to the level of supply reliability, it is necessary to quantify reliability in monetary terms. In reliability cost and worth analyses of power systems, the reliability-worth experienced by customers is compared with the cost incurred by the grid owner [1]. Customer interruption cost (CIC) is used as a substitute in the assessment of reliability-worth in electric power systems [2]. Numerous studies have been conducted to provide estimates of CICs and a wide range of methodologies has evolved. However, the use of different probability distribution functions (PDFs) to model CIC for planning and operating reliability-worth studies is uncommon.

Reliability-worth indices are determined for a given system or component and it is the interpretation of these indices that sheds light on how reliable the system is. Most reliability cost and worth analyses in previous research use average values for the input parameters and present the reliability-worth outputs as estimates of the mean values. Using average values for the input and output parameters ignore the shape of the parameter PDF. Several indices have been proposed for reliability-worth studies (e.g. expected CIC

(ECOST), interrupted energy assessment rate (IEAR) and cost of unserved energy (CUE)). The selection and definition of these indices are very much dependent on the methodologies used and the purpose of the study. To estimate consequences for the customers, the reliability-worth index ECOST is computed and presented in this paper. The work presented in this paper was carried out on a power distribution network.

Several techniques have been developed for use in evaluation of reliability-worth indices of a given power system. The techniques can however be grouped as either deterministic or probabilistic. Deterministic (also termed analytical) techniques have been used for many years in reliability-worth analyses of radial distribution systems to calculate the average load point reliability-worth indices [3]. The average load point reliability-worth indices are estimated using a mathematical model that uses average input parameter values (e.g. repair time, switching time, CIC values, etc.). They are limited for the work proposed in this paper because it is almost impossible to apply these techniques when non-constant parameter inputs are considered.

Probabilistic techniques have advantage over deterministic techniques in that they are able to account for the stochastic behaviour of power networks [4,5]. The main probabilistic techniques are simulations, the most important being Monte-Carlo simulations (MCSs). The time sequential MCS plays an important role in the work presented here because it takes into account the stochastic nature of power systems in a chronological order. This approach allows for the inclusion of the time dimension in the reliability-worth analysis [6]. The inputs of a reliability analysis, such as component failure rates, restoration times and CIC values, are treated as random rather than average values and are allowed to

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take on values according to chosen PDF [7,8]. The performance of time sequential MCS is independent of the size of the network being analysed.

For the purpose of this research, all cost are given in South African Rand (R). 1€ (Euro) is equivalent to R10 approximately.

## 2. Measuring CICs

Several studies have been conducted to provide estimates of CICs. There is however, no universal agreement on the appropriateness of methodologies applicable to particular situations nor on the interpretation of the results obtained, but some appear to be more acceptable and useful to the industry than others.

CICs are challenging to estimate since they are functions of many different factors [9]. The customer survey approach [10], in which customers are specifically interviewed, is regarded by many researchers as the most practical and reliable process to obtain these costs. The strength of the method is that customers are in the best position to know their own costs [11–13]. This is also supported by the results from both analytical and blackout case studies, which show that for interruption cost assessment to be realistic, the cost information should be customer specific [11]. However, the main drawback with survey methods is that the results are quite sensitive to the survey design and implementation [1,10].

The impact of a power interruption is defined by the interrupted activities due to the interruption [14]. Customers use electricity in different ways that characterises their sectors. Therefore, CICs are assessed by surveys for different customer sectors, usually according to a particular standardized industrial classification (SIC) [15–17]. For example, customers can be divided into: residential, industrial, governmental and public, agricultural, and commercial customers. To be able to quantify how disrupted activities affect the interruption cost, customer valuations of these effects are also needed. In customer surveys, these valuations are often included and made on an inconvenience scale [14,18].

With a customer survey, only the direct rather than indirect costs are collected. In direct costing methods, customers are asked to identify the impact of a particular hypothetical outage scenario and the associated costs [10,15]. Depending on whether social or economic costs are collected, different survey methods are used. For all customer sectors, less so for the residential sector, the direct costs mostly have an economic impact. Therefore, a direct costing method is recommended for these customer sectors [19]. Residential surveys use contingent valuation methods that are designed to capture more intangible costs such as inconveniences. In the contingent valuation methods, customers are asked to state how much they are 'willing to pay' (WTP) to avoid an outage or how much they are 'willing to accept' (WTA) in compensation for an outage. A direct costing method can also be applied to the residential sector. It is recommended that several different methods be used for the residential sector [19].

Performing a customer survey is a time-consuming and expensive task that requires a large effort to collect a sufficient data sample [10]. Interruption cost data derived from surveys therefore includes a small sample of the possible outage events. Commonly, only the interruption costs for a worst case scenario is surveyed for a few outage durations [20]. Customer surveys will always generate some "bad" data, such as unrealistically high cost estimates. Therefore statistical analyses of the raw data should be conducted before the data are used [10]. There are procedures for identifying outliers [21].

The costs incurred due to power supply interruptions can be presented as a function of outage duration, and when expressed in this form it is known as a customer damage function (CDF) [22]. The CDF can be determined for a group of customers belonging to particular sector. In these cases, the interruption cost versus

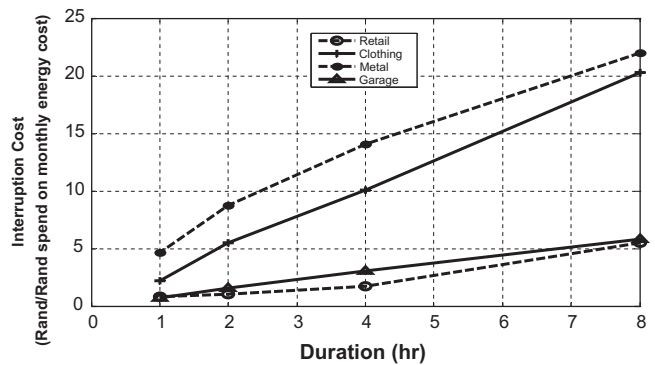


Fig. 1. Individual customer damage function models for different customer sectors [14].

duration plots are referred to as individual customer damage function (ICDF). ICDF are usually based on CIC data for the worst case scenario as shown in Fig. 1 [18].

Two different procedures for calculating the CDFs are: the average process and the aggregating process [23]. In the average process, the CIC data from the survey is first normalized. After normalization, an average value of the normalized cost for each customer sector and surveyed duration is calculated. The second procedure, the aggregating process, first summarizes the CIC data for each customer sector and duration. The result is then normalized by division of the summation of normalizing factors of each sector [10,20]. Common normalization factors are total annual electricity consumption, peak load or energy not supplied.

In Fig. 1, the normalization factor is average monthly energy cost and the unit of the ICDFs is therefore 'Rand' per 'Rand spent on monthly energy cost' [18,24,25]. The normalization process will give the values of the CDF marked with different symbols in Fig. 1. To estimate the CIC for any duration, linear interpolation is used between these values. Since the CIC data is only obtained for a worst case scenario, the CDF shows how the worst case cost varies with interruption duration.

The linearization of the costs with the duration of the interruption does not describe the dispersed nature of CIC that occurs for individual consumers as well as for the different durations [26,27]. It is therefore unrealistic to use average CIC values for the different durations considered and to assume the CIC value to have the same value 100% of the time. For realistic analyses, variability in CIC cannot be ignored and should be included in the model being used to represent it. Since PDFs allow for variation about the mean, they are a good tool for describing statistical variation (uncertainty) in the CIC modeling, from which the significance of including statistical variation in CIC modeling becomes clear.

Several PDFs have been identified for use in CIC analyses. Some include the Normal, Poisson, Weibull and Beta distributions [14,24,28]. However, relatively little work has been published on estimating reliability-worth indices associated with CIC derived from PDF. A number of multiplicative models have been applied to capture the time dependence of CIC. Studies show that the time dependencies in inputs are important when estimating the annual CIC, and ignoring them may lead to different planning and operational decisions [29].

### 2.1. Application of PDFs to reliability-worth outputs

The reliability-worth indices of a power system are stochastic values dependent on a network's topology and operating philosophy and conditions. The average values show how reliable the system is on average, but it is interesting to investigate the risk of extreme

cases. The level of skewness of the PDF of an index is important when interpreting the index [30,31]. Without a PDF, extreme consequences of power interruptions on consumers are neglected. Even though the tail of a PDF represents events that occur very infrequently the consequences of these events may be severe and must be considered when a system is analysed. PDF can be used to show how quantifying the probability of expected interruption cost (ECOST) within certain boundaries is important in the assessment of system reliability-worth.

### 3. Selection of PDF

Most PDFs are limited in the shapes they can exhibit, and are therefore used for specific data sets. The normal PDF is the most frequently used in estimating CIC but has infinite negative and positive range. In Refs. [26,27], it was concluded that it is impossible to model CIC data using a PDF which exhibit the same shape like the Normal distribution. This is because of the skewness of the CIC data in some of the studied durations. The aim is not to complicate the CIC data analysis by using different PDFs for different durations, but to make sure the CIC data is estimated accurately in a very simple way. The Weibull and Gamma distributions exhibit different shapes, however, their limitation is that they do not have a finite positive range which the beta PDF have [30,31]. Extensive research has been carried out, using the beta PDF. For example, in South Africa, it has been used to perform voltage drop calculations along distribution feeders. It is now the prescribed method for residential LV-feeder analysis in South Africa [32]. As far as CIC analyses are concerned, only preliminary work has been done to investigate the appropriateness of the PDF in CIC data fitting [24]. In this study the beta PDF was therefore used to model the CIC data.

The beta distribution describes the distribution of a random variable that lies within the interval (0,1) [33] and is defined by expressions (1) and (2). The beta distribution is very versatile in the shapes it can take. Fig. 2 illustrates some of these, given different values of its shape parameters. The distribution has a finite range and the data can be scaled using the maximum value of the data set or some greater value.

$$F(x) = \frac{X^{\alpha-1}(1-X)^{\beta-1}}{B(\alpha,\beta)} \quad (1)$$

for  $0 \leq X \leq 1$ ,  $\alpha > 0$  and  $\beta > 0$  where

$$B(\alpha,\beta) = \int_0^1 X^{\alpha-1}(1-X)^{\beta-1} dX \quad (2)$$

where  $\alpha$  and  $\beta$  are the shape parameters.

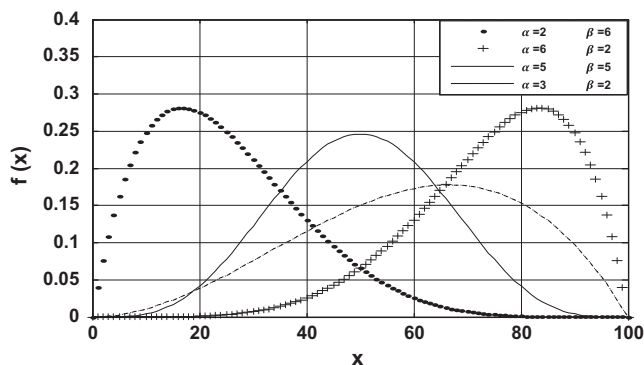


Fig. 2. Different shapes shown by beta PDF for different shape parameters,  $\alpha$  and  $\beta$ .

Various methods have been developed for estimating the parameters of the beta PDF. The most commonly used have been the method of moments, maximum likelihood method and program evaluation and review technique (PERT). From Ref. [33], both the method of moments and maximum likelihood were found to be the most efficient in determining the beta PDF parameters. The maximum likelihood methods requires extensive analysis or iterations to get the beta parameters. The method of moments was preferred over the maximum likelihood method because of its simple way of calculating the beta parameters when the mean and standard deviation values are available. Therefore, the method of moments has been used in the present study. Given the first moment about the origin or the mean ( $\mu$ ) and the second moment about the mean or the standard deviation ( $\sigma$ ) of a data set, the shape parameters of the corresponding beta distribution, for a given scaling factor  $C$ , can be computed using expressions (3) and (4).

$$\alpha = \frac{C\mu - \mu^2 - \sigma^2}{C\sigma^2} \quad (3)$$

$$\beta = \frac{(C - \mu)(C\mu - \mu^2 - \sigma^2)}{C\sigma^2} \quad (4)$$

For more information on the beta distribution and how to derive the above equations the reader is referred to Refs. [30,33]. The beta PDFs provide the input data and output for reliability-worth analysis as illustrated in the following small case study.

### 4. Case study

The RBTS [34] is an educational test system developed by the Power System Research Group at the University of Saskatchewan. The small radial distribution power system network used in the case study is taken from RBTS Bus 2:Feeder 3 as shown in Fig. 3. The power system network consists of one breaker F3 on the 11 kV side of a 33/11 kV transformer.

It has six 11/0.4 kV transformers, T1 to T6, one at each load point. These transformers have fuses that prevent transformer failure to affect the rest of the power system network. At each T-junction or branch isolators are located on both sides of lines and breakers, which enables the isolation of these components. The analysis considers unreliability caused by failure of station transformers, main breaker and distribution lines. The system is assumed to be in steady state such that the effect of failure of protection devices is neglected.

The reliability data used in the analysis is presented in Table 1 and is taken from [34]. The PDF for the time to failure was assumed to be exponential with the failure rate given in Table 1. The failure rate of overhead lines is calculated at 0.065 failures per year per kilometer. The PDF for load point interruption durations (repair/replacement time, switching time) is assumed to be lognormal [7].

The load model used in this paper is given in terms of the variation of average monthly energy cost for the different segments investigated and was adopted from similar work as in Refs. [24,25]. A working month of 30 days was assumed for all months. Table 2 presents the load point data used for the reliability-worth analysis in this paper.

In this paper a multiplicative approach with time varying cost factors for modeling temporal variations in CIC is taken. The temporal variations of CIC with time of day, day of week and month are modeled using two time-varying cost weight factors. This approach is described in detail in [14,24,29]. In this paper the influence on CIC due to time of day and day of week are combined and modeled using the time of day/day of week weight factor,  $f_{h/d}$  while the influence due to month of year is modeled using the month factor,  $f_m$ . The normalized CIC for customer segments

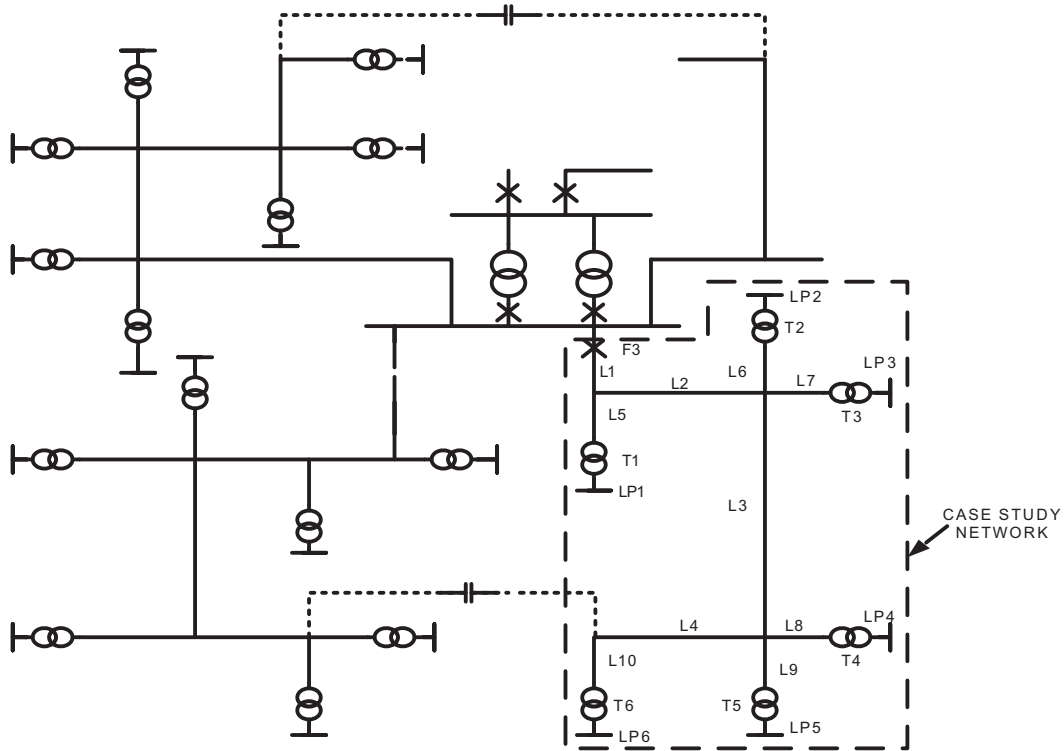


Fig. 3. One line diagram of the RBTS – Bus 2.

Table 1  
RBTS component reliability parameters.

Component	Description	Failure rate per year (f/yr)	Replacement/repair time (h)	Switching time (h)	Distribution
Transformer	11/0.4 kV	0.015	10.0 (1)	1.00 (0.4)	Lognormal
Breaker	11 kV	0.006	4.0 (0.4)	1.00 (0.4)	Lognormal
Transmission lines	L1 and L8	0.065	5.0 (1)	1.00 (0.4)	Lognormal
	L2, L3 and L10	0.182	5.0 (1)	1.00 (0.4)	Lognormal
	L4 and L7	0.117	5.0 (1)	1.00 (0.4)	Lognormal
	L5, L6 and L9	0.0975	5.0 (1)	1.00 (0.4)	Lognormal

The standard deviation of the distribution is given in brackets.

Table 2  
Load point data.

Load point	Number of customers	Customer sector	Average monthly energy cost [Rand]
LP1	50	Commercial (retail)	2200
LP2	100	Industrial (garage)	2000
LP3	40	Industrial (clothing)	1150
LP4	150	Commercial (retail)	2200
LP5	60	Industrial (metal)	4500
LP6	90	Commercial (retail)	2200
	50	Industrial (garage)	2000

due to power interruption of duration  $d$  occurring at time  $t$  is calculated as:

$$COST_A(t, d) = f_{h/d} f_m C_A(d) \tag{5}$$

where  $f_{h/d}$  is the time-varying cost weight factor for hourly deviation with respect to day of week from the reference time for customer segment  $A$ ,  $f_m$  is the time-varying cost weight factor for monthly deviation from reference time for customer segment  $A$ ,  $C_A(d)$  is the normalized reference (worst case) interruption cost for customer segment  $A$  due to a power interruption of duration  $d$ .

Both cost weight factors model the deviation of power interruption from the surveyed reference outage event. When a power interruption occurs at the reference time both cost weight factors are equal to unity, and the power interruption cost  $COST(t, d)$ , equals  $C(d)$ . The reference cost  $C(d)$  can either be modeled using the CDF approach or a PDF approach that captures the dispersion in the cost data. The ICDF models in Fig. 1 were used to estimate the CIC values. Uncertainty was applied to the CIC values of each

Table 3  
Beta parameters for summer weekday morning outage cost for the different segments.

Customer sector	Duration (scaling factor)			
	1 (30)	2 (50)	4 (100)	8 (150)
Retail	2.301 (81.881)	4.056 (194.782)	2.301 (131.458)	2.142 (56.070)
Clothing	2.090 (26.405)	2.024 (16.444)	1.864 (16.624)	1.474 (9.417)
Metal	0.295 (1.610)	0.146 (0.688)	0.288 (1.758)	0.219 (1.273)
Garage	1.165 (46.069)	1.527 (47.421)	1.161 (36.780)	2.392 (59.250)

NB: Beta parameters are given as  $\alpha (\beta)$ , where  $\beta$  value is in brackets.

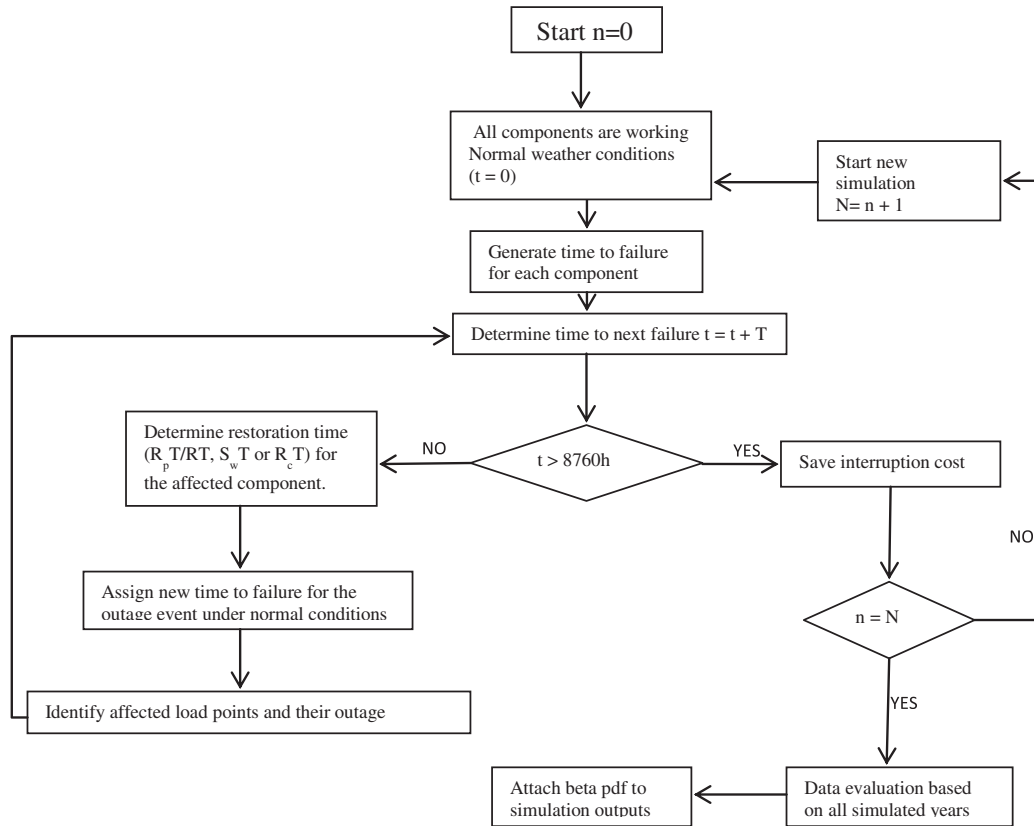


Fig. 4. Flowchart for the time sequential Monte-Carlo simulation.

customer sector by selecting random values from a specific beta PDF using the parameters given in Table 3 and time variation in CIC was applied by including the cost weight factors [24] in the analysis.

#### 4.1. Time-sequential Monte-Carlo simulation technique

The process used to assess the CIC of the RBTS is the time-sequential Monte-Carlo simulation technique. The algorithm in Ref. [24] is used in the analysis. Fig. 4 shows the detailed steps in the form of a flow chart. The simulation algorithm used to analyse the RBTS was developed using MATLAB®.

A base case analysis was first carried out on the network that considers the use of the ICDF to model CIC for each sector. The main aim of such an analysis is to provide a set of values for comparison with subsequent tests. In both analysis cases, a beta PDF was derived from the index computed. This aimed to capture any skewness in the distribution of the index.

## 5. Results

The 50 and 90-percentile values were determined from the performance index PDFs. A 90 percentile value represents the value of an index, such that risk of the actual value being above it is 10%. A 50-percentile value allows for a 50–50 chance for variation above and below the actual value and, for both a Normal and beta PDF, this value would correspond to the median of the data being described. Figs. 5–8 show the shapes of the ECOST index PDFs derived from the base case analysis and also from the analysis with time variation and uncertainty considered. It is clear from the shapes of the PDFs, corresponding to the base case, that there exists inherent skewness in the performance indices. Using average values to represent parameters neglects the distribution of the given

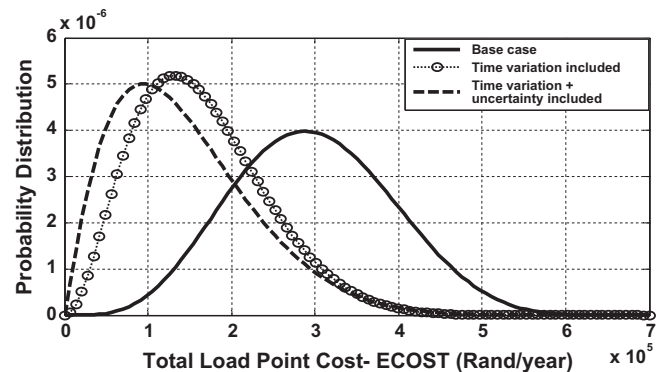


Fig. 5. Probability distribution of the ECOST for LP2.

parameters and excludes information on the variance and skewness of the resultant index. Table 4 shows the 50 and 90-percentile values of the six load point indices computed. Considering the base case values, a significant difference in the index values is noted in comparison with the results from the other two scenarios. This is attributed to the inclusion of time varying cost factors in estimating CIC. The values of the index reduce after inclusion of variability to the analysis. The percentage differences in 50 and 90-percentile values for each index are presented in Table 5.

#### 5.1. Interpretation of results

There is a significant difference in the ECOST values between the base case and the corresponding data sets. The 27–58% reduction in the ECOST index values after the inclusion of time variation in CIC values indicate how sensitive the index is to variability in reliability-worth inputs. The significant difference observed is



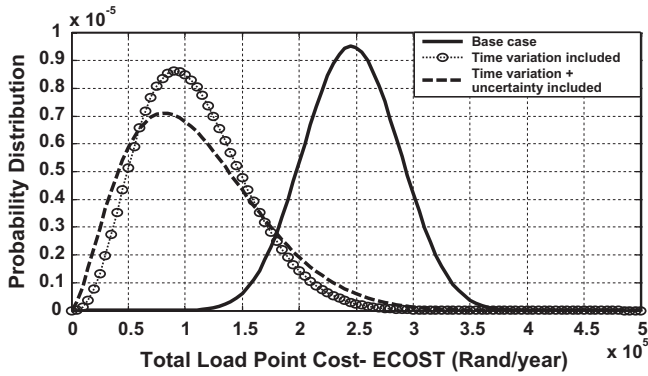


Fig. 6. Probability distribution of the ECOST for LP3.

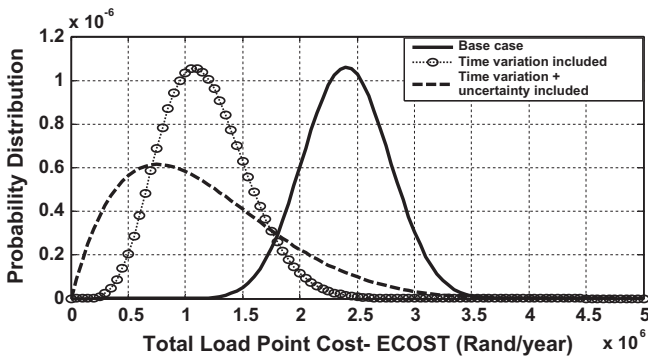


Fig. 7. Probability distribution of the ECOST for LP5.

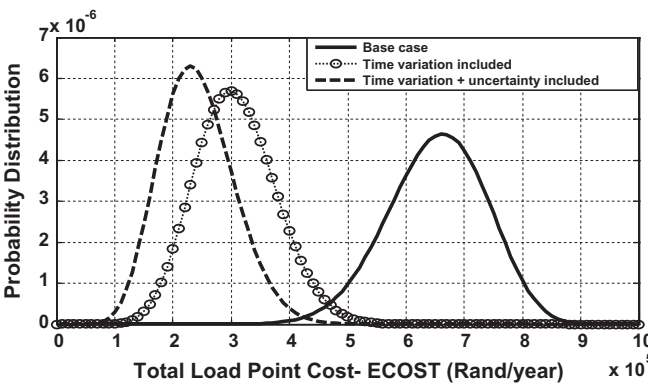


Fig. 8. Probability distribution of the ECOST for LP6.

indicative of a need for inclusion of time variation in CIC modeling. Common reliability-worth analyses assume CIC values to be constant for the different times of occurrence of power interruptions of the same duration. As mentioned earlier, the activities of different customers contribute to the variation of CIC values. Moreover, the impact of power interruptions on customers not only depends

on the activities being interrupted but is also time dependent. The analysis presented in this paper considered constant CIC values for different times of occurrence of power interruptions of the same duration – base case scenario. The reduction in index values presented in Table 4 shows how the impact of time of occurrence of power interruptions can be erroneously excluded in reliability-worth analyses, and ignoring it may lead to different planning and operational decisions. The time-line should be realistic and consider only the period of power interruption and its time of occurrence.

From Figs. 5–8, it is clear that the application of uncertainty to the reliability-worth input (CIC), changes the level of skewness of the PDFs. The fact that continuous PDFs were applied when describing reliability-worth inputs should therefore be noted. Average values have limited application when time dependent variability is considered. Planners and utility owners usually have to determine the level of network reinforcement and the cost attached to each alternative. Realistic and accurate reliability-worth analyses are critical to such decision making. Reliability cost and worth analyses are used to determine where in a power grid the reliability-worth (ECOST) exceeds cost of electric supply (reliability cost). These decisions are based on index values that, as observed from the results, have risk levels attached. Using average values means planners and utilities allows for a 50% risk level on the values used. In many cases, this is not good enough, such as for implementation of efficient energy delivery techniques. For effectiveness of these techniques, PDFs provide more information. This might include comparing low risk (high confidence) index values with the high risk (low confidence) index values to justify network reinforcements. For example, from Table 4, the ECOST results with uncertainty included indicate a 29.88% increase in ECOST index if one decides to use 10% risk values over 50% risk values. The increase in the risk cost can then be compared to the cost of implementing a equipment upgrade. This translates the justification into a rate of return analysis balancing capital expenditure against reducing the impact of failures, presenting a financial case that plant management is familiar with. The smaller the percentage in cost difference, the easier it is to justify investment in the network.

Another issue ignored by the use of average values is the likelihood of occurrence of extreme events. This can however be computed by analyzing the tails of PDFs. It is clear that the ECOST PDFs presented in this analysis with uncertainty considered are right skewed. The two extremes for this PDF are high and low values for ECOST. For example, the PDF of LP5 implies that low values of ECOST are more likely compared to high ECOST values. Comparing with PDFs of LP6, the likelihood of very low ECOST is significantly lower. However, it should be noted that while the values of the ECOST index reduced in this analysis, application of different statistical variations could cause different effects on the load point index and therefore, on the overall system index. Load point connection of customers with different characteristics e.g. ownership of backup power supply, may lead to different ECOST distributions. Since reliability-worth indices are sensitive to variability in the inputs as was shown by the results of this analysis, it is expected that indices computed could either increase or, as was the case in this analysis, decrease. The level of change is dependent on the system being analysed and the variability applied.

Table 4  
Results obtained from the Pdf of the six load point performance index (kRand).

Risk value	LP1 (%)		LP2 (%)		LP3 (%)		LP4 (%)		LP5 (%)		LP6 (%)	
	50	10	50	10	50	10	50	10	50	10	50	10
Base case	97.16	127.17	298.08	424.49	244.93	312.40	482.16	614.48	2436.80	3067.50	658.47	796.50
Time variation included	42.19	72.07	163.69	270.07	109.85	186.82	202.86	322.91	1196.00	1809.00	308.45	419.83
Time variation + uncertainty included	41.16	75.76	160.06	261.17	108.07	196.15	204.60	351.66	1152.90	2210.40	244.18	348.24

**Table 5**  
Percentage differences in 50 percentile and 90 percentile values for all load points.

	LP1	LP2	LP3	LP4	LP5	LP6
Base case	0.2360	0.2978	0.2160	0.2153	0.2056	0.1733
Time variation included	0.4146	0.3939	0.4120	0.3718	0.3389	0.2653
Time variation + uncertainty included	0.4567	0.3871	0.4490	0.4182	0.4784	0.2988

## 6. Conclusions

Different sections of power systems are exposed to various risks that must be accounted for when carrying out reliability-worth analysis on the network. The work presented in this paper is ongoing. A preliminary analysis was carried out using time sequential MCS on a test system (RBTS). It showed there is a need to move from the conventional use of average values to PDFs. Not only do PDFs account for various impact levels of power interruptions in power system networks, thus allowing for day to day reliability-worth analyses, they also enhance the interpretation of the index computed. It was found that ignoring time variations in CICs and taking the average values only, can severely underestimate the risks of extreme (high and low) CIC values. The technique and results obtained from the analysis demonstrate that reliability-worth indices can be predicted with a defined level of confidence. Expressing the results in this way will allow non-engineering managers to make meaningful managerial decisions about enhancing power system infrastructure and back-up. It will also assist regulators to determine rewards and penalties necessary to balance cost or tariffs against reliability.

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