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Decentralized electricity system sizing and placement in distribution networks

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ABSTRACT

A rapid method for sizing and placing of distributed electricity generation (DES) systems in an electric transmission network in respect to voltage has been developed and successfully validated. The new tool presented is in particularly useful for avoiding overvoltage situations, which are critical for the whole electricity system. The results show that DES placement closer to the transformer side is always more beneficial in terms of voltage than at the end of the line. Depending on the size of the DES unit, both up and downstream flow of power may occur. The method can be used for investigating a range of different placement and sizing configurations.

1. Introduction

The markets of decentralized electricity systems (DES) such as wind and solar power grow fast though their contribution to all electricity production is still marginal. However, their share of future electricity production may grow significantly as shown by many energy scenarios [1,2]. On a long-term basis by 2050, solar and wind power could represent close to 40% of global electricity production as a part of the climate change mitigation measures [2,3]. Locally or regionally, decentralized electricity could potentially represent even higher shares of all electricity. System integration issues and grid operation will therefore be central questions for these energy sources.

The power quality of decentralized small-scale electricity production imposes challenges to operation of the electric grid, and in particular for organically grown and step-wise built electricity distribution networks. Known problems include induced harmonics, voltage flicker, system reliability and voltage fluctuation due to large amounts of DES [5–11,16,17,19,24,25]. One of the most critical parameters limiting massive introduction of distributed energy generation in an already existing network is indeed the possible overvoltage that arises when supply exceeds consumption. The focus of this paper is on the voltage issue as this could be a potential source for physical damage and a restriction for large scale DES penetration.

The main aim of this work is to develop a fast tool to assess and visualize the voltage effects of decentralized energy units in a distribution network. Such a tool could be useful, e.g. for planners, designers and technology providers. The new algorithm to be presented enables easy and accurate sizing and positioning of DES units in an electricity distribution network.

Previous studies on voltage and decentralized power production has been reported, e.g. in [5–8,17,18]. The emphasis of these was mainly on safety, control, losses and grid reliability issues especially on fault situations. The overvoltage situations were considered in connection with some technical problems, e.g. islanding or single-line-to-ground fault and how decentralized power could help the grid to maintain its functionality in these situations [10,11]. Detailed investigation of the voltage issue is often based on sophisticated numerical grid simulations [21]. Willis and Scott [20] viewed the integration from an economical perspective. Voltage and power quality issues have been specifically discussed in [9–14], but often in qualitative terms. The authors were not aware of straightforward tools to generate voltage-profiles such as here. The closest resemblance of our model is the so-called one-line DC model [23].

The method in this paper starts from a known load pattern in the grid to produce a modified steady-state voltage profile when introducing DES. Though the method is static, the dynamic behavior of the electric system can be accessed through a point by point calculation over time. Basically a DES unit causes a disturbance in the voltage throughout the line. Of particular interest is to keep the possible overvoltage created within allowed limits. For example, in a 20 kV medium voltage distribution grid the voltage tolerance is approximately ±2% [22]. This could be achieved by limiting temporal DES production, re-positioning DES units, through DSM measures or electrical storage. The method is capable of handling all

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Nomenclature

a	power consumption density
b	discrete power production
I	current (A)
L	length of the transmission line
l	specific length
P	power (W)
p	position of DES unit
R	resistance (Ω)
U	voltage (V)
U_t	voltage at *i*th node (V)
ΔU	voltage difference (V)
x	distance (km)
X	reactance (Ω)
$\varDelta$	small variation in a variable
δ	Dirac delta function
θ	heaviside step function
φ	phase angle

Subscript

i	node
p	power (W)
d	density

Abbreviations

AC	alternating current
DC	direct current
DES	decentralized electricity systems
DSM	demand side management

these options and thus enables to locate the DES units more accurately in the grid. The method is also validated against a more sophisticated numerical network simulation program and continuous results.

2. Description of the model

There are several detailed tools for electricity network simulation such as MATPOWER [4] or DESIGEN [21]. Mostly these tools are suitable for sophisticated network analysis and require comprehensive data. Network effects for any arbitrary case can be solved accurately with such network simulation tools. However, these models are not suitable for high level placement analysis due to their rigid and inflexible structure.

Our method is fast and simple to use. The main input consists of average data on the cable properties, electric load, and production. The more sophisticated MATPOWER tool needs about the same kind of information on the electricity system, but making spatial changes in the system configuration, e.g. changing the place of the distributed production units, requires major and tedious efforts. In our model, the load and production nodes can easily be varied. The network is described as a linear line with only one connection to main line, e.g. transformer. The main line has an order of magnitude lower resistance (low impedance) than the distribution network so that the connection point can be approximated to remain at constant voltage in all load situations. Fig. 1 illustrates the linear network, nodes, and the connection point.

The method takes advantage of the orientation property of a loopless network. In a loopless network, the cables between adjacent nodes have an unambiguous orientation: the upstream node looks always toward the transformer and downstream node toward the end of the line. Reversing the power flow does not change this property. If the network is looped, it is no longer simply connected. This disables the use of current method. However, a loopless branched network can be approximated with a single line network by matching downstream consumption and impedance at each node. The voltage change between two nodes can be calculated with the well-known approximation for AC circuits [15]

$$\Delta U \approx I(R\cos\varphi + X\sin\varphi) = \frac{P}{U}(R + X\tan\varphi) \tag{1}$$

where ΔU = voltage difference between consecutive nodes, I = current (A), $(R + X\ \tan\varphi)$ = impedance (Ω), and P = power transmitted between nodes (W), R = resistance (Ω), X = reactance (Ω), and φ = phase angle.

A good approximation for analyzing voltage changes in a long line is to assume an evenly distributed load along the line. The transmission line is then divided into a set of discrete nodes (i). The discrete voltage change is obtained from Eq. (1) as $\Delta U = \mathrm{U}_{i+1} - U_i$. This yields a recursive formula for the voltage at the next node U_{i+1}, which depends on transmission power, cable impedance and voltage at previous node, as follows

$$U_{i+1} \approx \frac{1}{2}\left(U_i + \sqrt{U_i^2 - 4P_{\mathrm{Downstream}}(R + X\tan\varphi)_{i\to i+1}}\right) \tag{2}$$

where i is the ith node in the line, $P_{\mathrm{Downstream}}$ = power consumption on a downstream part of the line, and $(R + X\ \tan\varphi)_{i\to i+1}$ is the impedance of the transmission line between nodes i and $i + 1$.

The above approximation takes into account the voltage differences occurring over transmission line, but not over the individual loads. As the power flow in the transmission cable is the dominant factor contributing to voltage changes, the inaccuracy caused by neglecting the load voltage drops is normally very small, but could cause a minor deviation at the end of the line.

Determining the downstream power consumption/production in Eq. (2) is unambiguous for a loopless network. The power that is transmitted in the cable is the power needed to fill the net consumption (consumption minus production) downstream from the node in question. If the production exceeds the consumption downstream, then the voltage change ΔU between two nodes will change its sign and the voltage drop becomes a voltage jump. Net consumption of the downstream network includes all losses in the transmission cables. An asymmetric production/consumption pattern would cause some inaccuracy by increasing internal currents, but these are quite small in practice and can be neglected here. As the focus of this work is in large penetration of distributed power generation, some sort of even distribution of DES units can also be assumed.

Using the above methodology the position of DES units can be parameterized and optimum placement can be sought by minimizing the voltage peaks. This feature does not appear, to our knowledge, in conventional network calculation tools.

3. Validation

To validate the algorithm presented above, a comparison to MATPOWER and a continuous solution is derived. Two test cases are considered: (i) a transmission line of arbitrary length L with a constant power consumption density without DES and (ii) same as (i) but with DES in the middle of line. In the first case, the trans-

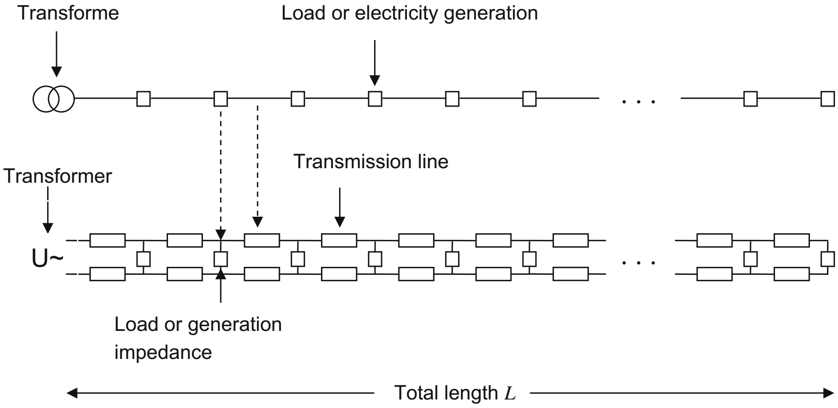

Fig. 1. Illustration of a single line transmission network used in this study.

mission power becomes simply a diminishing function. These cases where chosen to also allow an exact solution to be derived for the voltage.

To make the validation against an exact mathematical solution of voltage possible, a continuous formula is derived in the next. Let us assume a constant consumption density of a (W/m) and a discrete generation of size b (W) so that $aL = b$. The generation is located at point p which is at the middle of the line at $L/2$ (Fig. 2).

Modifying Eq. (1) for infinitesimal spatial steps, the voltage $U(x)$ at the line can be solved from following equation:

$$\frac{dU}{dx} = -\frac{(R + X\tan\varphi)}{l}\frac{T_p(x)}{U} \tag{3}$$

where $T_p(x)$ = transmission power as a function of the spatial coordinate x.

The power consumption is continuous and DES units are represented as delta functions. Power flow at each point is the total consumption of the remaining part of the line. Transmission power T_p is the integral of net power/consumption density P_d in the line from point x in question to the end of the line:

$$T_p(x) = \int_x^L P_d(x')dx' \tag{4}$$

Eq. (3) can be separated and its integral yields

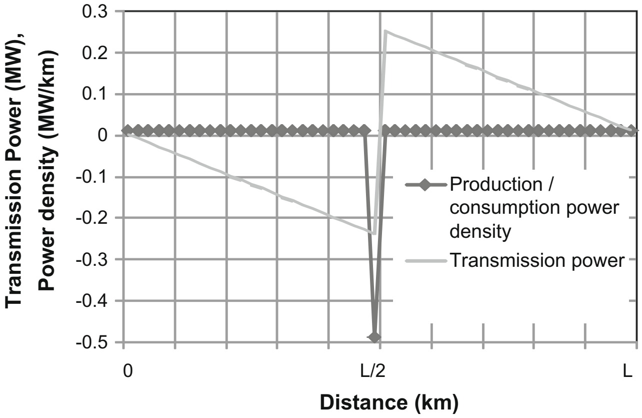

Fig. 2. Transmission power and consumption/production power along the line. Total production at $x = L/2$ equals to the total consumption over the line and therefore no transfer at transformer. Negative transmission power means opposite direction of power flow compared to normal. Net power is consumed at every node except one at $L/2$, which is net producer of power. L is the total length of the transmission line.

$$\int_{U_0}^{U(x)} U'dU' = \frac{1}{2}(U^2(x) - U_0^2) = -\frac{(R + X\tan\varphi)}{l}\int_0^x T_p(x')dx' \tag{5}$$

The specific impedance $(R + X \tan\varphi)/l$ is assumed constant and is therefore not included in integration. The transmission power T_p consists of the constant consumption density a and the discrete DES generation at $x = p$. The DES can be described with a Dirac's delta function $\delta(x)$ of size b which yields a Heaviside step function $\theta(x)$ when integrated

$$T_p(x) = \int_x^L [a - b\delta(x' - p)]dx' = a(L - x) - b[1 - \theta(x - p)] + C \tag{6}$$

As the transmission power at $x = L$ must be 0, it follows that $C = 0$.

Carrying out the integration of Eq. (6) gives

$$\begin{aligned}\int_0^x T_p(x')dx' &= \int_0^x a(L - x') - b[1 - \theta(x' - p)]dx' \\ &= \begin{cases}(aL - b)x - \frac{1}{2}ax^2, & 0 \leqslant x \leqslant p \\ ax\left(L - \frac{1}{2}x\right) + pb, & p < x \leqslant L\end{cases}\end{aligned} \tag{7}$$

Finally, combining Eqs. (3), (5), and (7) gives a continuous expression for the voltage as follows:

$$U(x) = \begin{cases}\sqrt{U_0^2 - 2\frac{(R+X\tan\varphi)}{l}\left[(aL - b)x - \frac{1}{2}ax^2\right]}, & 0 \leqslant x \leqslant p \\ \sqrt{U_0^2 - 2\frac{(R+X\tan\varphi)}{l}\left[ax\left(L - \frac{1}{2}x\right) + pb\right]}, & p < x \leqslant L\end{cases} \tag{8}$$

Comparing the discrete voltage model in Eq. (2) to the continuous one in Eq. (8) and to results from the numerical MATPOWER model, one can assess the accuracy and usefulness of the discrete model for placement of DES units in a transmission line.

The physical line parameters in the validation for a medium-voltage transmission line with $L = 100$ km, $a = 0.4$ MW/100 km, $U = 20$ kV and $R/L = 0.145\ \Omega$/km, $X/L = 0.302\ \Omega$/km, and $\tan\varphi = 0.10$. The validation results for the case without DES are shown in Fig. 3. All three models predict the voltage in a similar way as shown in Fig. 3a. and the inaccuracy of the new model is very small, or $\frac{\Delta U}{U} \leqslant 0.01\%$. The differences of the models over the line length $[0, L]$ against the network simulation results are shown in Fig. 3b. MATPOWER and the continuous model represent accurate results and hence the absolute differences are negligible (<0.001%). The discrete model based on voltage discretization at each node shows a larger divergence against MATPOWER but is still less than 0.005%.

The case with a single DES unit in the middle of the line ($x = L/2$) is shown in Fig. 4. The results from the three models are close to each other. The inaccuracy of the discrete model compared to other

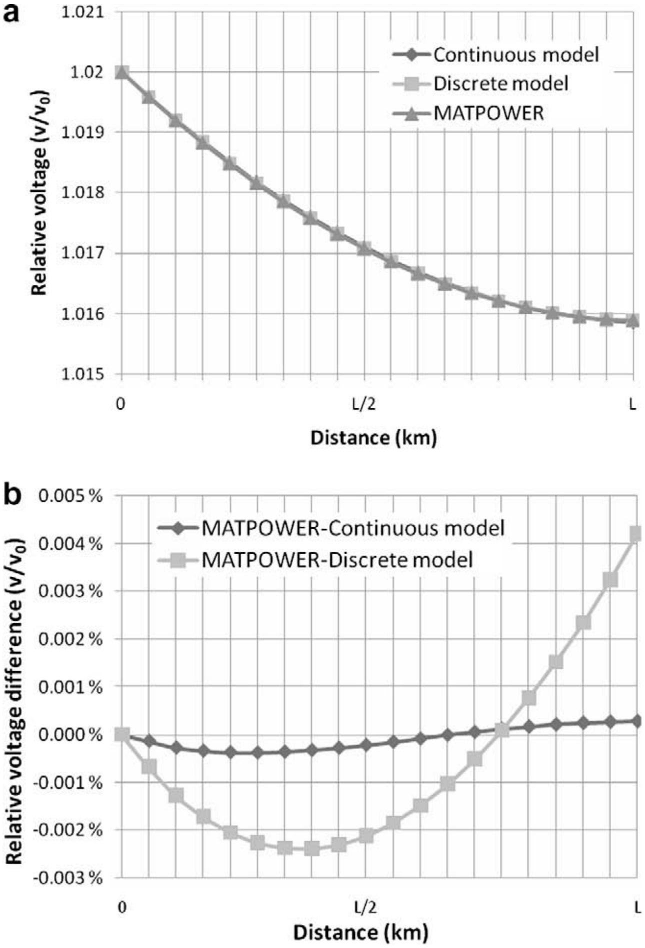

Fig. 3. Validation of the model without DES.

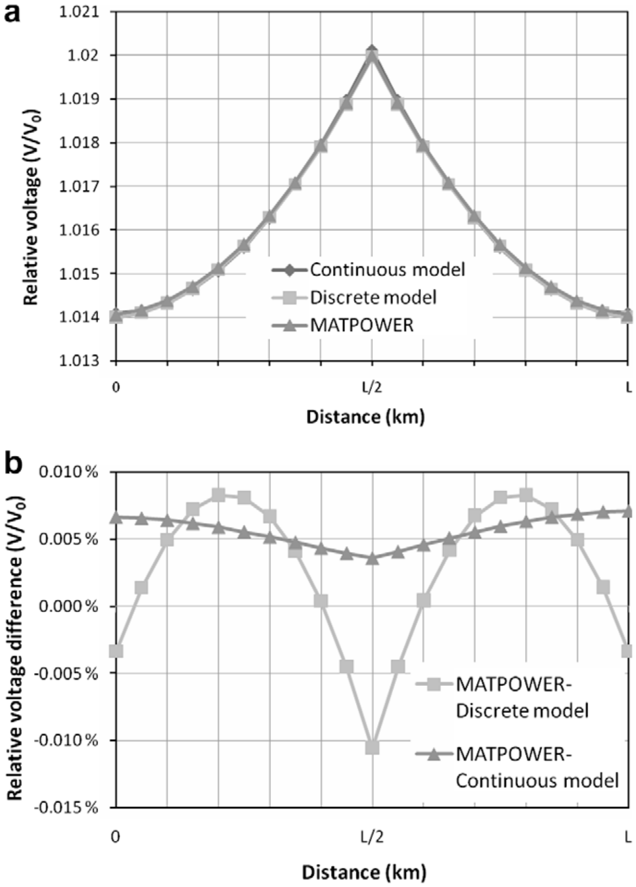

Fig. 4. Validation of the model with DES at $x = L/2$. L is the total length of the transmission line.

ones is now higher but still small, or $\frac{\Delta U}{U} \leqslant 0.01\%$. Fig. 4b illustrates the comparison of the models against the MATPOWER program. With the discrete model we observe the largest deviation (0.01%) at the source node ($x = L/2$) which is explained by the smaller node density used when discretizing the voltage. The number of nodes used in the discrete model was $N = 20$ and a higher number would naturally reduce the deviation.

Summarizing the results of the validation, it can be claimed that the accuracy of the new model is good enough for investigating the placement strategies of DES in respect to the voltage. A typical simulation run with MATPOWER takes around 3–5 min and generating the input around 5–15 min depending on the complexity of the simulated system. Our model runs in less than few minutes and for the input a few minutes is necessary. Both MATPOWER and our modeling approach are able to handle arbitrary load and DES input profiles.

In the analysis to follow, we employ the discrete voltage model in Eq. (2) as it is very flexible and allows arbitrary positioning of the load and production units, whereas the continuous model in Eq. (8) would need modifications to cope with such cases. However, for the validation purpose and with the simples cases used, Eq. (8) was well justified. Moreover, the accuracy of the discrete model is adequate for the more detailed analyses.

4. Application of the model

Next the model is used to highlight the effect of the relative size and positioning DES units in a medium voltage (20 kV) network. We use here the discrete model from Eq. (2) which enables easier and more versatile DES placing than the continuous model. The accuracy of the discrete model is adequate as shown earlier in the Chapter 3. Eq. (8) was used here for model validation only. When adding the consumption and line impedance data to this, then the voltage can be unambiguously determined throughout the whole line.

Firstly, a DES unit of different size is placed in the middle of the line at $x = L/2$ and the instantaneous voltage pattern over the line is calculated with the model (Fig. 5). The consumption is assumed

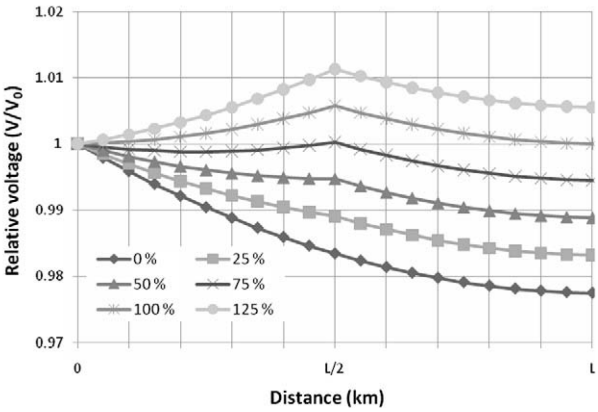

Fig. 5. Relative voltage change in a transmission line with length L. DES units are placed at $x = L/2$ and their size varies from 0% to 125% of the total consumption. L is the total length of the transmission line.

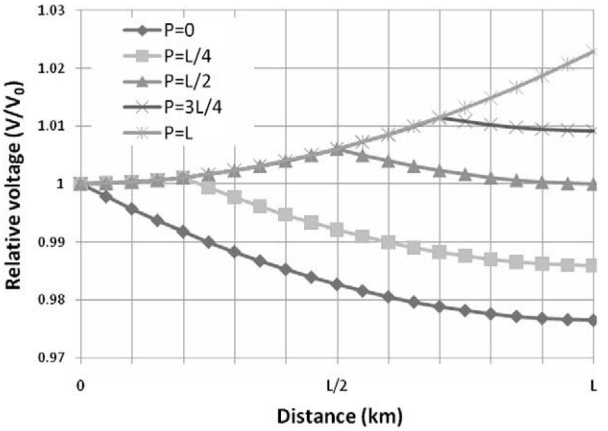

Fig. 6. Relative voltage change in a transmission line with length *L*. A DES unit of size 100% of total consumption is placed at different locations in the line.

constant over whole line. The DES unit size varies from 0% to 125% of the total consumption. Without any DES the voltage drops as expected. This is simply because the voltage drop is related to the transmission power which in turn approaches zero at the end of the line and hence also the voltage drop. Adding 25% of DES raises the voltage slightly throughout the line, but the form of the voltage remains almost the same as without DES. Increasing DES adequately (here >75% of load) a global maximum is observed at the point where the DES is placed. It should be observed that the portion of line from $x = L/2$ to $x = L$ consumes half of the power in the line yielding a declining voltage pattern. Going for a high DES share (>100%) in the network will increase the maximum voltage at $x = L/2$. Power from the DES unit will be transported both up and downstream from the generation point. With DES production corresponding to 100–125% of consumption, the voltage starts to increase at the transformer side since it will receive power from $x = L/2$ instead of transmitting electricity into the network. At a 100% DES share, the net production and consumption of electricity in the line is zero meaning that $dU/dx = 0$ at the transformer ($x = 0$). Exceeding 100% turns the network into a net producer of electricity.

Secondly, the effects of DES position in the network were investigated by placing a DES unit corresponding to 100% of the load at different points of the grid. Fig. 6 summarizes the results. If the DES unit is located at the transformer ($x = 0$), no changes are caused to the voltage profile. At the other extreme, placing the DES unit at the end of the line ($x = L$) yields largest overvoltage, which overrides the recommended upper limit of voltage change (2%). Actually the highest overvoltage is found at the same place where the DES unit is located. We may conclude that overvoltages with large amount of DES can be avoided through a proper placement strategy. In practice placing closer to the transformer side will reduce the voltage increase.

5. Conclusions

In this paper a quick method for sizing and placing of distributed electricity generation (DES) systems in an electric transmission network was presented. The main emphasis was in the affects of DES on grid voltage. Overvoltage may be very damaging and the tool can help to avoid such situations when employing large amounts of DES. The tool was successfully validated against a sophisticated numerical model.

As a general observation, placing DES units closer to the transformer side would enable integration of larger amount of DES than further away the line. Furthermore, depending on the size of DES unit both upstream and downstream flow of power may occur. If placing the DES units optimally, the share of DES could even exceed the load demand without adverse net effects, but the maximum amount of DES needs to be checked case by case due to the varying local conditions.

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