Control of Parallel-connected Inverters to Achieve Proportional Load Sharing

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Abstract In this paper, a completely new concept has been brought to the controller design for inverters. The capacitor C of the inverter LC filter is regarded as a part of the load and, hence, the control plant for the inverter controller is just the inductor L. This reduces the order of the control plant to be 1, reduces the variables to be measured for feedback to two, and considerably simplifies the design and analysis of the controller. The inherent limitations of the conventional droop control scheme are then revealed and it is proved that parallel-connected inverters should have the same per-unit impedance in order for them to share the load accurately in proportional to their power ratings if the conventional droop control scheme is adopted. The droop controllers should also generate the same voltage set-point for the inverters. Both of these two are impractical and difficult to meet, which results in errors in proportional load sharing. An improved droop controller is then proposed to achieve accurate proportional load sharing. It is robust against parameter drifts and component mismatches. The strategy also reduces the output voltage drop due to the effect of loading and droop control so that the output voltage can be maintained within the desired range around the rated value. Experimental results are provided to verify the analysis and design. *Copyright*^(C) 2011 IFAC.

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1. INTRODUCTION

Nowadays, more and more distributed generation and renewable energy sources, e.g. wind, solar and tidal power, are connected to the public grid via power inverters. They often form microgrids before being connected to the public grid (Guerrero et al., 2009). Due to the availability of high current power electronic devices, it is inevitable that several inverters are needed to be connected in parallel for high-power and/or low-cost applications. Another reason is that parallel-connected inverters provide system redundancy and high reliability, which is important for critical customers. A natural problem for parallel-connected inverters is that how the load is shared among them. A key technique is to use the droop control (Guerrero et al., 2005, 2007, 2010; Tuladhar et al., 1997; Brabandere et al., 2007; Zhong and Weiss, 2011), which is widely used in conventional power generation systems. The advantage is that no external communication mechanism is needed among the inverters. This enables good sharing for either linear or nonlinear loads (Guerrero et al., 2005; Tuladhar et al., 1997; Borup et al., 2001; Tuladhar et al., 2000).

The equal sharing of linear and nonlinear loads has been intensively investigated (Guerrero et al., 2005, 2007; Borup et al., 2001) and high accuracy of equal sharing can be achieved. A voltage bandwidth droop control was used to share nonlinear loads in (Tuladhar et al., 1997) and a small signal injection method was proposed to improve the reactive power sharing accuracy in (Tuladhar et al., 2000), which can also be extended to harmonic current sharing. It is pointed out in (Guerrero et al., 2005) that the output impedance of the inverters plays a critical role in power sharing and a droop controller for inverters with resistive output impedances is proposed for sharing linear and nonlinear loads (Guerrero et al., 2007).

Although significant progress has been made for the equal sharing of linear and nonlinear loads, it is still a problem to share loads accurately in proportional to the power ratings of the inverters. In particular, the accuracy of reactive power sharing (for the Q - E and $P - \omega$ droop) is not high (Li and Kao, 2009). Moreover, some approaches developed for equal sharing cannot be directly applied to proportional sharing. Another issue is that the output voltage drops due to the increase of the load and also due to the droop control. Hence, the proportional sharing problem needs to be investigated in a systematical way.

It has been recognised that adding an integral action to the droop controller is able to improve the accuracy of load sharing for grid-connected inverters; see (Marwali et al., 2004; Li et al., 2004; Dai et al., 2008). However, it is still a problem for inverters operated in the standalone mode and also there is an issue associated with the change of the operation mode. A strategy, which involves adding a virtual inductor and estimating the effect of the line impedance, was proposed in (Li and Kao, 2009) to improve the situation but the strategy is quite complicated and there is still room for improvements. All these strategies are sensitive to numerical computational errors, parameter drifts and component mismatches, to the best knowledge of the author. In this paper, it is proved that, in order for the parallel-connected inverters to share the load in proportional to their power ratings, the inverters should have the same per-unit impedance. It also requires that the RMS voltage set-points for the inverters to be the same. Both are very strong conditions. A robust droop controller is then proposed to achieve accurate proportional load sharing among inverters connected in parallel in microgrids operated in the standalone mode. The accuracy of sharing is no longer dependent on the output impedance of the inverters originally designed nor on the RMS voltage setpoint. Moreover, the controller is able to regulate the output voltage to reduce the effect of the load and droop control on the output voltage. In this paper, the robust droop controller is proposed for inverters with resistive output impedances and it can be applied to inverters with inductive output impedances as well, by using the Q-Eand $P - \omega$ droop.

An approach is also proposed in this paper to design an inverter to have a resistive output impedance. Since there is normally an LC filter in an inverter to reduce the switching noise in the output voltage, the approaches proposed in the literature all treat the LC filter as the control plant and the controllers are all designed based on this fact, to the best knowledge of the author. Most of them adopt the inductor current, the output current and the output voltage as feedback signals. Some adopt the capacitor current as feedback. In this paper, a completely new concept is brought to the controller design for inverters, based on the observation that the capacitor can be regarded as a part of the load, instead of a part of the control plant. Hence, the controller can be designed according to the filter inductor only. This reduces the order of the control plant to one and simplifies the control design and system analysis. Moreover, only two sensors (for the inductor current and the output voltage) are needed for feedback, which reduces the cost of the controller. Because of this, the inverter can be designed to have resistive output impedance over a wide range of frequencies, which considerably facilitates the sharing of nonlinear loads. The combination of the above leads to a very neat strategy to achieve accurate proportional load sharing.

2. CONTROLLER DESIGN FOR INDIVIDUAL INVERTERS TO ACHIEVE RESISTIVE OUTPUT IMPEDANCE

The circuit of a single-phase inverter under consideration is shown in Fig. 1(a). It consists of a single-phase H-bridge inverter powered by a DC source, and an LC filter. The inverter is connected to the AC bus via a circuit breaker CB and the load is assumed to be connected to the AC bus. The control signal u is converted to a PWM signal to drive the H-bridge so that the average of u_f over a switching period is the same as u, i.e. $u \approx u_f$. Hence, the PWM block and the H-bridge can and will be ignored in the controller design. The inductor current i is measured to construct a controller so that the output impedance of the inverter is forced to be resistive and that it dominates the impedance between the inverter and the AC bus. Moreover, the output voltage v_o is measured, together with the inductor current i, for proportional load sharing. This avoids measuring the load current i_o and reduces the cost and complexity of the controller.

As is now well known, it is advantageous to force the output impedance of parallel-connected inverters to be resistive (Guerrero et al., 2005). The inverter consists of an LC filter and, to the best knowledge of the author, all control strategies proposed in the literature have adopted a second-order model for the inverter. Here, an important step forward has been made, that is to regard the capacitor C as a part of the load instead of a part of the inverter. This reduces the control plant to an H-bridge and an inductor, as shown in Figure 1(b). The advantages of this are: 1) it reduces the order of the control plant to be 1; 2) it reduces the signals to be measured for feedback to one(excluding the feedback for voltage/power control); and 3) it considerably simplifies the design and analysis of the controller, which facilitates the understanding of the nature of inverter control.

Since the control plant is now of the first order, the controller can be designed with ease. The proposed controller, as shown in Figure 2, involves the feedback of the inductor current i with a proportional gain.



(b) Used for controller design

Figure 1. A singe-phase inverter



Figure 2. The proposed controller to achieve a resistive output impedance

The following two equations hold for the closed-loop system consisting of Figure 1(b) and Figure 2:

$$u = v_r - K_i i,$$

$$u_f = sLi + v_o.$$

Since the average of u_f over a switching period is the same as u, there is (approximately)

$$v_r - K_i i = sLi + v_o,$$

which gives

with

$$v_o = v_r - Z_o\left(s\right) \cdot i$$

$$Z_o\left(s\right) = sL + K_i.$$

If the gain K_i is chosen big enough, the effect of the inductance is not significant and the output impedance can be made nearly purely resistive over a wide range of frequencies. Then, the output impedance is roughly

$$Z_o(s) \approx K_i,$$

which is independent of the inductance.

With the above control strategy, the inverter can be approximated as a controlled ideal voltage supply v_r cascaded with a resistive output impedance R_o described as

$$v_o = v_r - R_o i \tag{1}$$

with

$$R_o = K_i$$
.

Note that $v_o \approx u = v_r$ if no load is connected.

3. INHERENT LIMITATIONS OF THE CONVENTIONAL DROOP CONTROL SCHEME

Fig. 3 shows two inverters with resistive output impedances connected in parallel. The line impedances are omitted because the output impedances of the inverters are designed to dominate the impedance from the inverter to the AC-bus. The reference voltages of the two inverters are, respectively,

$$v_{r1} = \sqrt{2E_1}\sin(\omega_1 t + \delta_1),$$

$$v_{r2} = \sqrt{2E_2}\sin(\omega_2 t + \delta_2).$$

The power ratings of the inverters are $S_1^* = E^* I_1^*$ and $S_2^* = E^* I_2^*$. They share the same output voltage v_o .

Since the output impedances of the inverters are designed to be resistive (constant) over a wide range of frequencies, all the harmonic current components can be shared among the inverters in proportion to their power ratings. Hence, proportional sharing can be achieved for linear and nonlinear loads and the following analysis is applicable to both linear and nonlinear loads.



Figure 3. Two inverters with resistive output impedances connected in parallel

The active and reactive power of each inverter injected into the bus (Guerrero et al., 2005) are



Figure 4. The conventional droop control scheme

$$P_i = \frac{E_i V_o \cos \delta_i - V_o^2}{R_{oi}},\tag{2}$$

$$Q_i = -\frac{E_i V_o}{R_{oi}} \sin \delta_i. \tag{3}$$

In order for the inverters to share the load, the conventional droop controller

$$E_i = E^* - n_i P_i, \tag{4}$$

$$\omega_i = \omega^* + m_i Q_i, \tag{5}$$

as shown in Figure 4, is widely used to generate the amplitude and frequency of the voltage reference v_{ri} for each inverter (Guerrero et al., 2007), where ω^* is the rated frequency. Note that the P - E and $Q - \omega$ droop is used because the output impedances are resistive. Otherwise, the $P-\omega$ and Q-E droop should be used when the output impedances are inductive. The drooping coefficients n_i and m_i are normally determined by the desired voltage and frequency drops, respectively, at the rated active power and reactive power. The frequency ω_i is integrated to form the phase of the voltage reference v_{ri} .

In order for the inverters to share the load in proportional to their power ratings, the droop coefficients of the inverters should be in inverse proportional to their power ratings (Tuladhar et al., 1997), i.e., n_i and m_i should be chosen to satisfy

$$n_1 S_1^* = n_2 S_2^* = \dots = n_n S_n^*, \tag{6}$$

 $m_1 S_1^* = m_2 S_2^* = \dots = m_n S_n^*.$ (7)

It is easy to see that n_i and m_i also satisfy $\frac{n_1}{m_1} = \frac{n_2}{m_2} = \dots = \frac{n_n}{m_n}.$

3.1 Active power sharing

Substituting (4) into (2), the active power of the two inverters can be obtained as

$$P_i = \frac{E^* \cos \delta_i - V_o}{n_i \cos \delta_i + R_{oi}/V_o}.$$
(8)

Substituting (8) into (4), the voltage amplitude deviation of the two inverters is

$$\Delta E = E_2 - E_1 = \frac{E^* \cos \delta_1 - V_o}{\cos \delta_1 + \frac{R_{o1}}{n_1 V_o}} - \frac{E^* \cos \delta_2 - V_o}{\cos \delta_2 + \frac{R_{o2}}{n_2 V_o}}.$$
 (9)

It is known from (Tuladhar et al., 1997) that the voltage deviation of the two units leads to considerable errors in load sharing. In order for

$$n_1 P_1 = n_2 P_2$$
 or $\frac{P_1}{S_1^*} = \frac{P_2}{S_2^*}$

to hold, the voltage deviation ΔE should be 0 according to (4). This is a very strict condition because there are always numerical computational errors, disturbances, parameter drifts and component mismatches. This condition is satisfied if

$$\frac{n_1}{R_{o1}} = \frac{n_2}{R_{o2}} \tag{10}$$

and

$$\delta_1 = \delta_2. \tag{11}$$

In other words, n_i should be chosen to be proportional to its output impedance R_{oi} .

Taking (6) into account, in order to achieve accurate sharing of active power, the (resistive) output impedance should be designed to satisfy

$$R_{o1}S_1^* = R_{o2}S_2^* = \dots = R_{on}S_n^*.$$
 (12)

Since the per-unit output impedance of Inverter i is

$$\gamma_i = \frac{R_{oi}}{E^*/I_i^*} = \frac{R_{oi}S_i^*}{(E^*)^2},$$

there is

$$\gamma_1 = \gamma_2 = \dots = \gamma_n \doteq \gamma.$$

This simply means that the per-unit output impedances of all inverters should be the same in order to achieve accurate proportional active power sharing. Recall that power transformers with different power ratings have more or less the same per-unit output impedances (although not resistive).

3.2 Reactive power sharing

When the system is in the steady state, the two inverters work under the same frequency, i.e., $\omega_1 = \omega_2$. It is well know that this guarantees the accuracy of reactive power sharing for inverters with resistive output impedances (or the accuracy of active power sharing for inverters with inductive output impedances); see e.g. (Li and Kao, 2009). Indeed, from (5), there is

$$m_1Q_1 = m_2Q_2.$$

Since the coefficients m_i are chosen to satisfy (7), reactive power sharing proportional to their power ratings is (always) achieved, i.e.,

$$\frac{Q_1}{S_1^*} = \frac{Q_2}{S_2^*}.$$

According to (3), there is

$$m_1 \frac{E_1 V_o}{R_{o1}} \sin \delta_1 = m_2 \frac{E_2 V_o}{R_{o2}} \sin \delta_2.$$
(13)

If $\delta_1 = \delta_2$ and $E_1 = E_2$ then $\frac{m_1}{R_{o1}} = \frac{m_2}{R_{o2}}.$ (14)

Theorem For inverters designed to have resistive output impedances, if the system is stable, then the following two sets of conditions are equivalent:

$$\begin{cases} E_1 = E_2\\ \frac{n_1}{R_{o1}} = \frac{n_2}{R_{o2}} \iff \begin{cases} \delta_1 = \delta_2\\ \frac{m_1}{R_{o1}} = \frac{m_2}{R_{o2}} \end{cases}.$$

Proof. If (10) and $E_1 = E_2$ hold, then proportional active power sharing is achieved according to (4). As a result, (11) holds according to (9) and (13). Furthermore, reactive power sharing proportional to their ratings is achieved and (14) holds. Conversely, if (11) and (14) hold, then $E_1 = E_2$ according to (13). Furthermore, (10) holds according to (9). This completes the proof.

This theorem indicates that if inverters with resistive output impedances are designed to achieve accurate proportional active power sharing, then they also achieve proportional reactive power sharing in the ideal case. The converse is also true. However, it is almost impossible in reality if this strategy is used. It is difficult to maintain $E_1 = E_2 = \cdots = E_n$ because there are always numerical computational errors, disturbances and noises. It is also difficult to maintain $\gamma_1 = \gamma_2 = \cdots = \gamma_n$ because of parameter drifts and component mismatches. A mechanism is needed to guarantee that accurate proportional load sharing can be achieved.

4. ROBUST DROOP CONTROLLER TO ACHIEVE ACCURATE PROPORTIONAL LOAD SHARING

As a matter of fact, the voltage droop (4) can be re-written as

$$\Delta E_i = E_i - E^* = -n_i P_i,$$

and the voltage E_i can be implemented via integrating ΔE_i , that is,

$$E_i = \int_0^t \Delta E_i \mathrm{d}t.$$

This works for the grid-connected mode where ΔE_i is eventually 0 (so that the desired power is sent to the grid without error), as proposed in (Marwali et al., 2004; Li et al., 2004; Dai et al., 2008). However, it does not work for the standalone mode because the actual power P_i is determined by the load and ΔE_i cannot be 0. This is why different controllers had to be used for the standalone mode and the grid-connected mode.

According to (1), the output voltage v_o drops when the load increases. It also drops due to the droop control, according to (4). In order to make sure that the output voltage remains within a certain required range, the output voltage drop $E^* - V_o$ can be added to ΔE_i via an amplifier K_e . This actually results in an improved droop controller shown in Figure 5. It is able to eliminate (at least considerably reduce) the impact of computational errors, noises and disturbances. As to be explained below, it is also able to maintain accurate proportional load sharing and hence robust with respect to parameter drifts, component mismatches and disturbances. In the steady state, the input to the integrator should be 0. Hence,

$$n_i P_i = K_e (E^* - V_o).$$
 (15)

The right-hand side of the above equation is always the same for all inverters connected in parallel as long as K_e is chosen the same, which can be easily met. Hence, accurate real power sharing can be achieved without having the same E_i , which is more natural. The active power sharing

is no longer dependent on the inverter output impedances and is also immune to the numerical computational errors and disturbances, which guarantees the accuracy of real power sharing. Moreover, from (15), there is

$$V_o = E^* - \frac{n_i}{K_e} P_i.$$

The output voltage drop is no longer determined by the output impedance originally designed but by the parameters n_i , K_e and the actual power P_i . It can be considerably reduced by using a large K_e . This easily solves the compromise between the voltage drop and the speed of dynamic responses (Tuladhar et al., 1997). The droop coefficient n_i can be chosen big to speed up the dynamic response while the voltage drop can be kept small by using a large K_e . Although the output impedance of Inverter *i* is initially designed as R_{oi} , e.g., as designed using the approach presented before, which could be big, the overall voltage drop could be made small.



Figure 5. The proposed robust droop controller to obtain accurate proportional load sharing

5. EXPERIMENTAL RESULTS

The above strategy has been verified in a laboratory setup. It consists of two single-phase inverters controlled by dSPACE kits and powered by separate 42V DC power supplies. The values of the inductors and capacitors are 2.35mH and 22μ F, respectively. The switching frequency is 7.5kHz and the frequency of the system is 50Hz. The nominal output voltage is 12V RMS and $K_e = 10$. The droop coefficients are: $n_1 = 0.4$ and $n_2 = 0.8$; $m_1 = 0.1$ and $m_2 = 0.2$. Hence, it is expected that $P_1 = 2P_2$. In the experiments, K_i was chosen as 4 for both inverters. Due to the configuration of the hardware setup, the voltage for Inverter 2 was measured by the controller of Inverter 1 and then sent out via a DAC channel, which was then sampled by the controller of Inverter 2. This brought some latency into the system but the effect was not noticeable.

5.1 With a linear load

A linear load of about 9Ω was connected to Inverter 2 initially. Inverter 1 was connected to the system at around t = 2 second and was then disconnected at around t = 7.5 second. Figure 6 shows the relevant curves of the



Figure 6. Experimental results with a linear load

experiment. It can be seen that the two inverters shared the load very accurately in the ratio of 2 : 1, although $E_1 \neq E_2$. A prominent feature is that the voltage E_1 and E_2 are different and both are higher than the rated value 12V. There was no need to change the operation mode of Inverter 2 when connecting or disconnecting Inverter 1.

5.2 With a nonlinear load

A nonlinear load, consisting of a rectifier loaded with an LC filter and the same rheostat used in the previous experiment, was connected to Inverter 2 initially. Inverter 1 was connected to the system at around t = 2.7 second and was then disconnected at around t = 9.7 second. Figure 7 shows the relevant curves of the experiment. It can be seen that the two inverters were still able to share the load very accurately in the ratio of 2 : 1, although $E_1 \neq E_2$. The dynamic performance did not change much either.

6. CONCLUSIONS

In this paper, it has been shown that the capacitor of the inverter LC filter can be regarded as a part of the load instead of a part of the inverter, which considerably reduces the complexity of system analysis and controller design and reduces the cost of the controller. The, the inherent limitations of the conventional droop control scheme has





Figure 7. Experimental results with a nonlinear load

been exposed. In order to achieve accurate proportional load sharing among parallel-connected inverters, the inverters should have the same per-unit resistive output impedances and the voltage set-point (E_i) should be the same. These are almost impossible in reality. An improved droop control strategy is then proposed to obtain accurate proportional load sharing for microgrids working in the standalone mode (and naturally also in the grid-connected mode). This strategy does not require that the voltage setpoints of the inverters to be the same; it does not require the output impedance to be the same either. The proposed strategy is also able to maintain excellent capability of voltage regulation. The strategy proposed here works for inverters with resistive output impedances but it can be applied to inverters with inductive output impedances by using the Q - E and $P - \omega$ droop.

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