

# The Simulation of an Induction Motor in Transient Behavior Taking into Account the Saturation Effect

Ileana Calomfirescu

"Politehnica" University of Bucharest, Bucharest, Romania, ileana.calomfirescu@yahoo.co.uk

**Abstract**— It is known the fact that for a digitally controlled induction machine, taking into account the saturation effect implies some unknown elements, like the nonlinearity of the magnetization curve. In this paper is studied the induction motor in transient behavior considering the saturation effect (the nonlinear expression between the magnetization inductance  $L_m$  and the magnetization current  $I_m$ ), [1]. This has been possible by creating an optimization program developed in Matlab, [2]. Also there had been used data from laboratory tests, [3], but because of the laboratory conditions the induction motor could not be used at the given rated values. The Matlab program is linked to another program, developed in Simulink, thus obtaining the electromagnetic torque, the stator and rotor currents in A-B-C three phase coordinates system and the stator and rotors currents in d-q orthogonal coordinates system, the mechanical velocity, the rotors current pulsation, the electrical slip. To begin, there had been used the Clark and Park transformations,[4], because the usage of constant orthogonal voltages simplifies the program. Then there were used the stator and rotors currents equations. For the mechanical part of the induction motor were calculated the viscosity coefficient and the inertial moment, [5] There had been created several subsystems in which was introduced the optimization made for the induction motor.

## I. INTRODUCTION

The modeling of an induction motor which takes into account the saturation effect has encountered difficulties because of the unknown electromagnetic parameters. This paper presents a model of the nonlinear induction motor in Matlab/Simulink programming environment, which has results that are very similar to the ones in the data sheet.

The nonlinearity of the induction motor is given by the magnetic saturation effect. The leakage inductances and the main inductances are highlighted by the relationship between the fluxes and the currents. This implies that the magnetization curve  $I_m(L_m)$  is used for an optimization for the nonlinear induction motor. If there are major perturbations of the stators frequency (2-3 times the value of the rated stator current), the leakage inductances are influenced by the saturation path of the main fluxes. Therefore it can be made a segregation between the two saturation aspects (the main flux path and the leakage flux path). It is customary that the leakage inductance are considered constant and equal ( $L_s=L_r$ ); if the saturation of the leakage paths is taken into account, only if the saturations paths are separated, the inductances are exclusively considered in relation with the currents windings.

The magnetization of the rotor core of the induction machine occurs in AC current at the stator and rotor, but at different frequencies. Also, the hysteresis phenomenon develops in a different manner from the rotor to the stator.

The skin effect of the induction motor is developed in the massive conductors and in the massive parts of the electric machine (deep rotor bars). The skin effect is influenced by the magnetic saturation of the main flux paths (massives parts) or the saturation of the leakage paths (the deep rotor bars). Including in the mathematical equations the two conditions, is possible to create a realistic model, but only through numerical methods of field.

To ensure equivalence between the real machine and the mathematical model there has been used the Park transformation in orthogonal coordinate system (meaning the inverse matrix is equal to the transposed matrix, therefore resulting the equivalence between the apparen powers). Also the equivalence between the real machine and the mathematical model must be satisfied from other points of view, such as the electromagnetic torque, the magnetic energy stored in the air gap, the leakage inductances and the main path inductances.

## II. ALGORITHM AND SOFTWARE IMPLEMENTATION

For the modeling of a nonlinear induction motor are used the equations of the voltage, as in the matrix form, equation (1), [6], as it follows:

$$[u_{srdq}] = [Z] \cdot [i_{srdq}] + [L] \cdot \frac{d}{dt} [i_{srdq}] \quad (1)$$

The next equations, (2), (3) and (4) are based on the orthogonal coordinates system d-q for the stator and rotor.

$$[u_{srdq}] = \begin{bmatrix} u_{sd} \\ u_{sq} \\ u_{rd} \\ u_{rq} \end{bmatrix} \text{ and } [i_{srdq}] = \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \quad (2)$$

$$[L] = \begin{bmatrix} (L_s + L_{md}) & L_{dq} & L_{md} & L_{dq} \\ L_{dq} & (L_s + L_{mq}) & L_{dq} & L_{mq} \\ L_{md} & L_{dq} & (L_r + L_{md}) & L_{dq} \\ L_{dq} & L_{mq} & L_{dq} & (L_r + L_{mq}) \end{bmatrix} \quad (3)$$

and

$$[Z] = \begin{bmatrix} R_s & -\omega_s(L_s + L_m) & 0 & -\omega_s L_m \\ \omega_s(L_s + L_m) & R_s & \omega_s L_m & 0 \\ 0 & \omega_s L_m & R_r & -\omega_r(L_r + L_m) \\ \omega_s L_m & 0 & \omega_r(L_r + L_m) & R_r \end{bmatrix} \quad (4)$$

$\omega_r$  and  $\omega_s$  are the angular velocities of the stator and respectively of the rotor.

These are the induction motor equations in saturated behavior, the Park transformation being in respect of the rotating field, in steady-state behavior.

The electromagnetic torque's form is as in equation (5):

$$M_{em} = p \cdot L_m \cdot (i_{sq} i_{rd} - i_{sd} i_{rd}). \quad (5)$$

The electromagnetic torque is used for the validation of the model for the induction motor, [7] and [8].

The coupling (magnetization) inductance,  $L_m$ , between the stator and the rotor of the induction motor has a sinusoidal variation in respect to the rotors position. This inductance has the following formula, as in equation (6), after the reducing at the stator:

$$L_m = \frac{3}{2} L_{0s}. \quad (6)$$

The iron losses occur only in the stator, except large variations of the rotors frequency.

### III. RESULTS

The simulation of the mathematical model in orthogonal d-q coordinate system of the nonlinear induction motor is developed in Matlab and Simulink programming environment. For the mathematical model of the induction motor was used Simulink programming environment, and in Matlab has been made an optimization for the magnetization curve. The necessary data for this optimization were obtained through the no-load test and drop speed test of the induction motor.

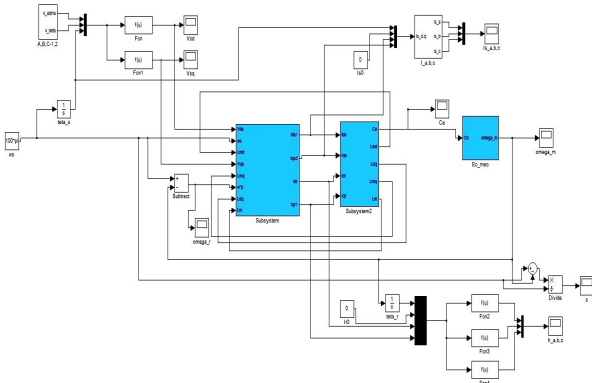


Fig. 1. The scheme of the nonlinear induction motor in Simulink

In figure 1, is presented the scheme of a nonlinear induction motor built in Simulink. For the calculation of the stator and rotor voltages in d-q coordinate system Park and Clark transformations were used in this scheme. It was necessary to perform an optimization for the magnetization curve, with the data obtained from a no-load test. The no-load test means that the induction machine works at synchronous speed  $\omega_r = \omega_s$  in steady-state behavior. To obtain a synchronous speed the machine is rotated with

an outside power engine, which has less than 10% of the induction motors power; the induction motor is fed from a variable voltage transformer or a synchronous generator alternator, which has a voltage with values between 0.05 and 1.25 of the rated voltage  $U_n$  of the induction motor. The iron losses can be taken into account by short-circuiting in a symmetric manner the stator winding on the d and q axes.

This test also permits the knowledge of the iron loss resistance's variation  $R_{fe}$  in respect to the magnetization current at a given frequency  $\omega_s$ . Generally speaking, the iron losses (from here it can be obtained the iron resistance in the stator winding) depend on the stators frequency. If the stators frequency,  $\omega_s$ , is modified and the phase voltage is also modified, collaborated with the fact that the synchronism speed is kept, all these lead to an estimation of the dependence of the iron losses resistance to  $\omega_s$  and  $I_{of}$  (represents the stator current on one phase for the no-load test). In the Park transformation, the flux is composed of the magnetization flux,  $\phi_m$ , and the leakage fluxes,  $\phi_{md}$  and  $\phi_{mq}$ .

In the Simulink program were found these quantities: the stator and rotors voltages for the orthogonal d-q coordinate system,  $U_{sd}$  and  $U_{sq}$ , the stator and rotors currents also in orthogonal d-q coordinate system,  $I_{sd}$ ,  $I_{sq}$ ,  $I_{rd}$  and  $I_{rq}$ , the stator and rotors currents in three phase A-B-C coordinate system, the electrical slip,  $s$ , the rotor currents pulsation,  $\omega_r$ , and the mechanical speed,  $\Omega$ .

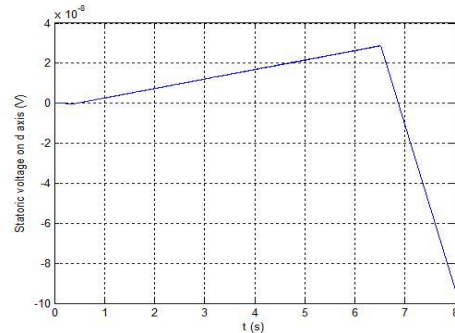


Fig. 2. The stators voltage on d axis.

The next plot, figure 2, represents the stator voltage on d axis,  $U_{sd}$ , which has a very small value, approximately  $10^{-8}$  V, what may be approximated to zero.

In the next graphic, figure 3, are represented the stator and rotor currents in d-q orthogonal coordinate system, which have also constants values, after the induction motor enters in the steady-state behavior.

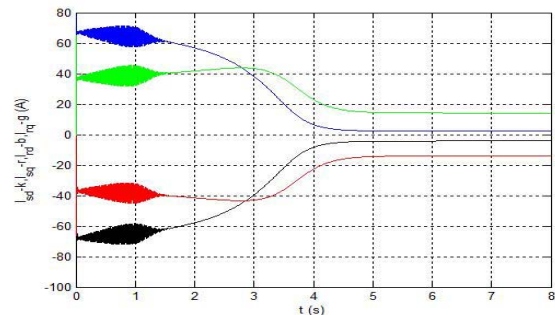


Fig. 3. The stator and rotors current in d-q coordinate system.

The plots that are represented in the figures 4 and 5 represent the stator and rotor currents in three phase coordinate system A-B-C,  $I_{sA}$ ,  $I_{sB}$ ,  $I_{sC}$ ,  $I_{rA}$ ,  $I_{rB}$ ,  $I_{rC}$ . This was possible because of the usage of the inverse Park transformation, starting with the d-q orthogonal currents, neglecting the homopolar component,  $I_{s0}$ , for the stator and also for the rotor,  $I_{r0}$ . The rotor pulsation is integrated to find the values of the rotor currents; the rotor pulsation has the following form:

$$\omega_r = \omega_s - \omega. \quad (7)$$

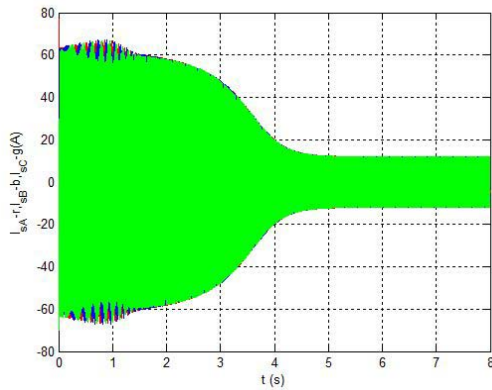


Fig. 4. The stators currents in A-B-C coordinates system.

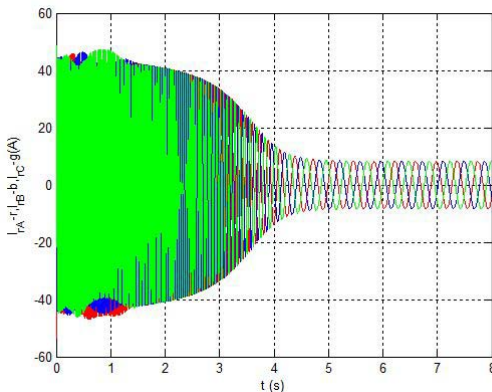


Fig. 5. The rotor currents in A-B-C coordinates system.

The rotor currents have a sinusoidal form, having an approximate period for the transient behavior of 4.5s. The stator current has an approximate value of 12A in steady-state behavior, while the rotor currents reach smaller values, around 8A, in the same behavior.

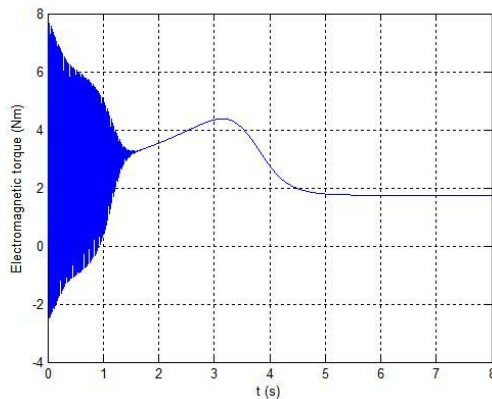


Fig. 6. The electromagnetic torque.

In the plot displayed in figure 6 shows the electromagnetic torque,  $M_{em}$ . It can be noticed the fact that the electromagnetic torque enters the steady-state behavior after 5.3 s, having a value of nearly 2 Nm, keeping in mind that the resistant torque is of 1 Nm. To calculate the electromagnetic torque were used algebraic loops of the stator and rotor currents. There also have been used algebraic loops for the calculation of the rotors pulsation, the leakage inductances of the stator and of the rotor, and last but not least the magnetization inductance.

In the graphic in figure 7, is presented the electrical slip of the induction motor, where it can be observed that the values for the electrical slip are between 1 and 0.05, confirming that the induction machine works as a motor.

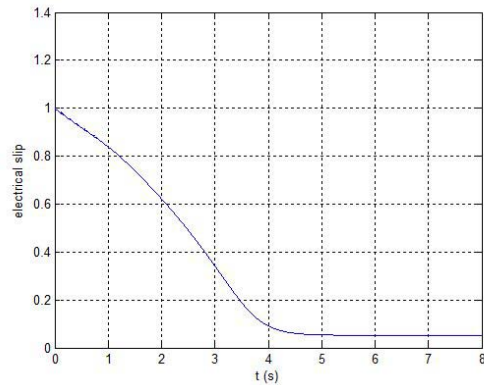


Fig. 7. The electrical slip.

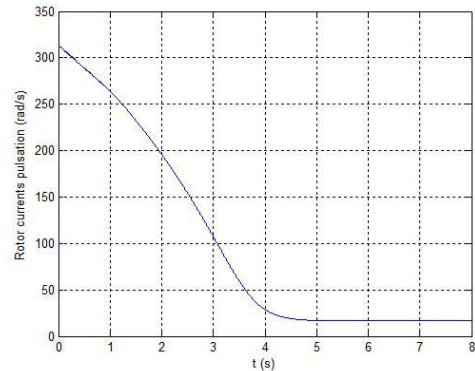


Fig. 8. The rotors currents pulsation.

In the plot that is presented in figure 8 is plotted the rotor currents pulsation,  $\omega_r$ , which has a value of approximately 17 rad/s after the transient behavior.

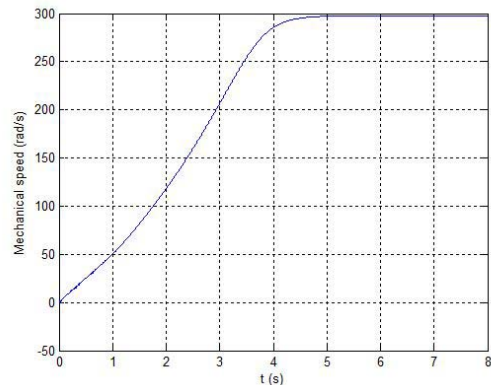


Fig. 9. The mechanical speed.

The mechanical speed is presented in figure 9, having a stable value at about 300 rad/s. The mechanical speed has been calculated with an algebraic loop, which takes into account the rotor currents pulsation.

The magnetization current,  $i_m$  has a mathematical expression that can be written in respect to the rotor and stator currents on d-q axes,  $i_{md}$  and  $i_{mq}$ .

$$i_{md} = i_{sd} + i_{rd}, \quad i_{mq} = i_{sq} + i_{rq}. \quad (8)$$

The magnetization current module can be written as it follows in equation (9):

$$|i_m| = \sqrt{(i_{md}^2 + i_{mq}^2)}. \quad (9)$$

Each of the magnetization fluxes on the d-q axes,  $\varphi_{md}$  and  $\varphi_{mq}$ , can be described by the following mathematical formulas, (10):

$$\varphi_{md} = L_m \cdot i_{md}, \quad \varphi_{mq} = L_m \cdot i_{mq}. \quad (10)$$

The magnetization inductance is noted with the symbol  $L_m$ , and it can be deduced with the help of the motor's magnetization curve. The magnetization flux,  $\varphi_m$ , of the induction motor without the resistant torque has the following mathematical expression, (11):

$$\varphi_m = \frac{|L_m|}{|i_m|}. \quad (11)$$

In the expression above  $L_m$  is the magnetization inductance and  $i_m$  represents the magnetization current.

For  $u_{sd} = \frac{d\varphi_{sd}}{dt}$  there is the next mathematical expression:

$$\frac{d\varphi_{sd}}{dt} = \frac{d}{dt}(L_{\sigma s} \cdot i_{sd} + \varphi_{md}). \quad (12)$$

If the leakage inductances,  $L_{\sigma s}$ , are assumed to be constants, the first expression of the  $u_{sd}$  becomes:

$$\frac{d\varphi_{sd}}{dt} = L_{\sigma s} \frac{di_{sd}}{dt} + \frac{d\varphi_{md}}{dt}. \quad (13)$$

The differential magnetization flux on d axis is in respect to the static magnetizing inductance:

$$\frac{d\varphi_{md}}{dt} = \frac{d}{dt}(L_m \cdot i_{md}). \quad (14)$$

The variation of the magnetization inductance  $L_m$  with the  $i_m$  module, can be written as follows:

$$\frac{d\varphi_{md}}{dt} = L_m \cdot \frac{di_{md}}{dt} + i_{md} \cdot \frac{dL_m}{dt} \quad (15)$$

$$\frac{d}{dt} \left( \sqrt{i_{md}^2 + i_{mq}^2} \right) = \frac{i_{md}}{i_m} \cdot \frac{d|i_{md}|}{dt} + \frac{i_{mq}}{|i_m|} \cdot \frac{di_{mq}}{dt}.$$

The differential magnetization flux on d axis is given by the following expression:

$$\frac{d\varphi_{md}}{dt} = L_m \cdot \frac{di_m}{dt} + i_{md} \cdot \frac{dL_m}{d|i_m|} \cdot \left( \frac{i_{md}}{|i_m|} \frac{di_{md}}{dt} + \frac{i_{mq}}{|i_m|} \frac{di_{mq}}{dt} \right). \quad (16)$$

By developing the terms in equation (16) is obtained the next expression:

$$\frac{d\varphi_{md}}{dt} = \left( L_m + \frac{dL_m}{d|i_m|} \cdot \frac{i_{md}^2}{|i_m|} \right) \frac{di_{md}}{dt} + \frac{dL_m}{d|i_m|} \cdot \frac{i_{md} \cdot i_{mq}}{|i_m|} \frac{di_{mq}}{dt} \quad (17)$$

It can be observed that the magnetization flux on the d axis depends on  $\frac{di_{md}}{dt}$  and  $\frac{di_{mq}}{dt}$ . The same remark it can be said for the flux on q axis:

$$L_{md} = L_m + L_{dq} \frac{i_{md}}{i_{mq}} \quad (18)$$

$$L_{mq} = L_m + L_{dq} \frac{i_{mq}}{i_{md}} \quad (19)$$

The inductances that appear in the above expressions,  $L_{md}$  and  $L_{mq}$ , are the magnetization inductances on d and q axes. In the linear behavior, these two inductances are equal to  $L_m$ . It can be said that calculation methods are a simplified model of the induction machine, where  $L_{dq}$  is the crossover inductance between the d and q axes, which is equal to zero if considered the linear model.

For the optimization were used the data provided by an oscilloscope; the oscilloscope was used for viewing the current curve at the no-load test. These data have been processed in a Matlab program, being achieved a third degree polynomial for the magnetization curve.

#### IV. CONCLUSIONS

In the present paper is desired the achievement of a realistic model for the induction motor, taking onto account the nonlinearity of the magnetization curve. This was made possible by creating an optimization model and identifying the magnetization inductance  $L_m$ . The optimization was made in Matlab/Simulink, using data that was retrieved in the context of laboratory tests for an induction motor. Some of this test is the no-load test to establish the magnetization curve and the DC test to determine the stator resistance. It has to be taken into consideration the fact that the test were made under laboratory conditions and the induction motor could not be used at the rated values given by the data sheet. Therefore the results

could be compared only in terms of shape, but not in terms of numerical values. The Matlab optimization can show the graph of the magnetization curve, after the data processing from the motor. The identification of the electromagnetic parameters of the induction motor be as accurate as possible.

#### ACKNOWLEDGMENT

The work has been co-funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labor, Family and Social Protection through the Financial Agreement Posdru/88 /1.5/s/61178.

#### REFERENCES

- [1] Ion Boldea, "Parametrii masinilor electrice, identificare, estimare si validare", ed. *Academiei Romane, Bucuresti*, pp 97-119, 125-140, 144-155, 1991.
- [2] Won Young Yang, Wenwu Cao, Tae-Sang Chung, John Morris, "Applied Numerical Methods using MATLAB", 1998
- [3] I. S. Gheorghiu, Al. S. Fransua, "Tratat de Masini electrice, vol al III-lea, Masini Asincrone", ed. *Academiei R. S. Romania, Bucuresti*, 1971.
- [4] D. O. Kisch, "Reglarea vectoriala a masinilor de curent alternativ", ed. *ICPE*, 2006.
- [5] M. Juffer, "Traité d'Electricité, Vol. 10, Machines électriques", *Press Polytechniques et Universitaires Romanades*, Paris, 1995, pp.384.
- [6] C. Ghita, "Modelare si parametrii convertoarelor electromecanice", ed. *Printech, Bucuresti*, pp 60-64,78, 89-97, 2003.
- [7] Mihai Iordache, Mihai Dogaru, Dragoş Niculae, Ileana Calomfirescu, "Computer – Aided Analysis of Induction Motor", *Annals of the University of Craiova, Electrical Engineering*, No. 34, 2010, pp. 224 - 228, ISSN 1842 – 4805.
- [8] M. Iordache, Ileana Calomfirescu, D. Niculae, M. Dogaru, „Simulation of Induction Motor Using State Equations”, *Proceeding of the 7<sup>th</sup> International Symposium Advanced Topics in Electrical Engineering – ATEE'11*, 12-14 May 2011 – Bucharest, ROMANIA, *Editura Politehnica Press*, pp. 59 - 64, ISSN: 2068-7966, Print ISBN: 978-1-4577-0507-6.