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Modified differential evolution algorithm for optimal power flow with non-smooth cost functions

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ABSTRACT

Differential evolution (DE) is a simple but powerful evolutionary optimization algorithm with continually outperforming many of the already existing stochastic and direct search global optimization techniques. DE algorithm is a new optimization method that can handle non-differentiable, non-linear, and multimodal objective functions. This paper presents an efficient modified differential evolution (MDE) algorithm for solving optimal power flow (OPF) with non-smooth and non-convex generator fuel cost curves. Modifications in mutation rule are suggested to the original DE algorithm, that enhance its rate of convergence with a better solution quality. A six-bus and the IEEE 30 bus test systems with three different types of generator cost curves are used for testing and validation purposes. Simulation results demonstrate that MDE algorithm provides very remarkable results compared to those reported recently in the literature.

1. Introduction

Optimal power flow (OPF) is one of the main tools for optimal operation and planning of modern power systems. The OPF is, hence, the basic tool that allows electric utilities to determine secure and economic operating conditions for an electric power system. An OPF adjusts the controllable quantities in the system to optimize an objective function, while satisfying a set of physical and operational constraints. This makes the OPF problem a large-scale highly non-linear constrained optimization problem.

The OPF problem has been solved via many traditional optimization methods such as linear programming, non-linear programming, quadratic programming, Newton-based techniques and interior point methods. A comprehensive review of various optimization techniques available in the literature is reported in Refs. [1,2]. Usually, these methods rely on the assumption that the fuel cost characteristic of a generating unit is a smooth, convex function. However, there are situations where it is not possible, or appropriate, to represent the unit's fuel cost characteristic as a convex function. For example, this situation arises when valve-points, unit prohibited operating zones, or multiple fuels are present. Hence, the true global optimum of the problem could not be reached easily. New numerical methods are then needed to cope with these difficulties, specially, those with high speed search to the optimal and not being trapped in local minima.

In recent years, many heuristic algorithms such as genetic algorithms (GA) [3,4], evolutionary programming (EP) [5,6], tabu search (TS) [7], particle swarm optimization (PSO) [8] and simulated annealing (SA) [9], have been proposed to solve the OPF problem, without any restrictions on the shape of the cost curves. The results reported were promising and encouraging for further research in this direction.

Recently, a new evolutionary computation technique, called differential evolution (DE), has been developed and introduced by Storn and Price [10]. DE algorithm is a stochastic population-based search method successfully applied in global optimization problems. DE combines simple arithmetic operators with the classical operators of crossover, mutation and selection to evolve from a randomly generated starting population to a final solution [11,12].

This paper presents an efficient modified differential evolution (MDE) algorithm for solving optimal power flow (OPF) with non-smooth cost functions. Modifications in mutation rule are suggested to the original DE algorithm that explores the solution space with a random localisation, enhancing its rate of convergence for a better solution quality. In order to demonstrate the suitability of the proposed approach, MDE algorithm was applied to the six-bus and IEEE 30 bus test systems with three different types of generator cost curves. Simulation results demonstrate that MDE algorithm is superior to the original DE and appears to be fast providing very remarkable results compared to those reported in the literature recently.

2. Optimal power flow problem formulation

The OPF problem is considered as a general minimization problem with constraints, and can be written in the following form:

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$$\text{Minimize} \quad f(x,u) \tag{1}$$
$$\text{Subject to}: \quad g(x,u) = 0 \tag{2}$$
$$h(x,u) \leqslant 0 \tag{3}$$

where $f(x,u)$ is the objective function, $g(x,u)$ is the equality constraints and represent typical load flow equations. $h(x,u)$ is the system operating constraints. x is the vector of state variables consisting of slack bus real power P_{G1}, load bus voltages V_L, generator reactive power outputs Q_G, and transmission line loading S_l. Therefore, x can be expressed as:

$$x^T = [P_{G1}, V_{L1} \dots V_{LNL}, Q_{G1} \dots Q_{GNG}, S_{l1} \dots S_{lNB}] \tag{4}$$

where NL, NG and NB are the number of load buses, the number of generators and the number of transmission lines, respectively.

U is the vector of control variables consisting of real power outputs P_G except at the slack bus, generator voltages V_G, transformer tap settings T. Hence, u can be expressed as:

$$U^T = [P_{G2} \dots P_{GNG}, V_{G1} \dots V_{GNG}, T_1 \dots T_{NT}] \tag{5}$$

where NT is the number of regulating transformers.

The objective function for the OPF reflects the cost associated with generating power in the system. The objective function for the entire power system can then be written as the sum of the fuel cost model for each generator:

$$f = \sum_{i=1}^{NG} f_i(\$/h) \tag{6}$$

where f_i is the fuel cost of the ith generator.

The system operating constraints $h(x, u)$ include:

(1) Generation constraints:
For stable operation, generator voltages, real power outputs and reactive power outputs are restricted by the lower and upper limits as follows:

$$V_{Gi}^{\min} \leqslant V_{Gi} \leqslant V_{Gi}^{\max}, \quad i \in NG, \tag{7}$$
$$P_{Gi}^{\min} \leqslant P_{Gi} \leqslant P_{Gi}^{\max}, \quad i \in NG, \tag{8}$$
$$Q_{Gi}^{\min} \leqslant Q_{Gi} \leqslant Q_{Gi}^{\max}, \quad i \in NG. \tag{9}$$

(2) Transformer constraints:
Transformer tap settings are restricted by the minimum and maximum limits as follows:

$$T_i^{\min} \leqslant T_i \leqslant T_i^{\max}, \quad i \in NT. \tag{10}$$

(3) Security constraints:
These incorporate the constraints of voltage magnitudes of load buses as well as transmission line loadings as follows:

$$V_{Li}^{\min} \leqslant V_{Li} \leqslant V_{Li}^{\max}, \quad i \in NL, \tag{11}$$
$$S_{li} \leqslant S_{li}^{\max}, \quad i \in NB. \tag{12}$$

3. Overview of differential evolution algorithm

Differential evolution (DE) is a relatively recent heuristic technique designed to optimize problems over continuous domains [10,11]. In DE, each decision variable is represented in the chromosome (individual) by a real number. As in any other evolutionary algorithm, the initial population of DE is randomly generated, and then evaluated. After that, the selection process takes place. During the selection stage, three parents are chosen and they generate a single offspring which competes with a parent to determine which one passes to the following generation. DE generates a single offspring (instead of two like in the genetic algorithm) by adding the weighted difference vector between two parents to a third parent. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector replaces the vector to which it was compared.

An optimization task consisting of D parameters can be presented by a D-dimensional vector. In DE, a population of N_P solution vectors is randomly created at the start. This population is successfully improved over G generations by applying mutation, crossover and selection operators, to reach an optimal solution [10,11]. The main steps of the DE algorithm are given below:

Initialization
Evaluation
Repeat
  Mutation
  Crossover
  Evaluation
  Selection
Until *(Termination criteria are met)*

3.1. Initialization

Typically, each decision parameter in every vector of the initial population is assigned a randomly chosen value from within its corresponding feasible bounds:

$$X_{j,i}^{(0)} = X_j^{\min} + \mu_j (X_j^{\max} - X_j^{\min}), \quad i = 1, \dots, N_P, \quad j = 1, \dots, D \tag{13}$$

where μ_j denotes a uniformly distributed random number within the range [0,1], generated anew for each value of j. $X_j^{\max}$ and $X_j^{\min}$ are the upper and lower bounds of the jth decision parameter, respectively.

3.2. Mutation

The mutation operator creates mutant vectors X_i' by perturbing a randomly selected vector X_a with the difference of two other randomly selected vectors X_b and X_c, according to the following expression:

$$X_i'^{(G)} = X_a^{(G)} + F(X_b^{(G)} - X_c^{(G)}), \quad i = 1, \dots, N_P \tag{14}$$

where a, b, and c are randomly chosen indices, such that a, b, $c \in \{1,\dots,N_P\}$ and $a \neq b \neq c \neq i$. It should be noted that new (random) values for a, b, and c have to be generated for each value of i. The scaling factor F is an algorithm control parameter in the range [0,2] which is used to adjust the perturbation size in the mutation operator and improve algorithm convergence.

3.3. Crossover

In order to increase the diversity among the mutant parameter vectors, crossover is introduced. To this end, a trial vector X_i'' is created from the components of each mutant vector X_i' and its corresponding target vector X_i, based on a series of D-1 binomial experiments of the following form:

$$X_{j,i}''^{(G)} = \begin{cases} X_{j,i}'^{(G)} & \text{if } \rho_j \leqslant C_R \text{ or } j = q \\ X_{j,i}^{(G)} & \text{otherwise}, \end{cases} \quad i = 1, \dots, N_P, \; j = 1, \dots, D \tag{15}$$

where ρ_j denotes a uniformly distributed random number within the range [0,1), generated anew for each value of j. The crossover constant C_R which is usually chosen from within the range [0,1], is an algorithm parameter that controls the diversity of the population and aids the algorithm to escape from local minima. q is a randomly chosen index $\in\{1,\ldots,D\}$, which is used to ensure that the trial vector gets at least one parameter from the mutant vector.

3.4. Selection

The selection operator forms the population by choosing between the trial vectors and their predecessors (target vectors) those individuals that present a better fitness or are more optimal according to (16).

$$X_i^{(G+1)} = \begin{cases} X_i''^{(G)} & \text{if } f(X_i''^{(G)}) \leqslant f(X_i^{(G)}) \\ X_i^{(G)}7 & \text{otherwise}, \end{cases} \quad i = 1,\ldots,N_P \tag{16}$$

This optimization process is repeated for several generations, allowing individuals to improve their fitness as they explore the solution space in search of optimal values.

Price and Storn proposed several variants of the basic DE which are denoted using the notation DE/x/y/z, where x refers to the perturbation type, y the number of pair of vectors used in the perturbation process and z the crossover scheme used in the recombination process. Nevertheless, one highly beneficial method that deserves special mention is the DE/best/2/bin which perturbs the best solution found so far with two difference vectors based on a binomial distribution crossover scheme:

$$X_i'^{(G)} = X_{\text{best}}^{(G)} + F(X_a^{(G)} - X_b^{(G)} + X_c^{(G)} + X_d^{(G)}), \quad i = 1,\ldots,N_P \tag{17}$$

where X_a, X_b, X_c and X_d are randomly chosen vectors from the set $\{1,\ldots,N_P\}$, mutually different and different to the target vector. X_a, X_b, X_c and X_d are generated anew for each parent vector. X_{best} is the best solution found so far in the optimization process. This strategy dramatically improves the convergence rate of the algorithm. However, in multimodal problems, this strategy could lead to premature convergence of the algorithm.

4. Modified differential evolution algorithm

This version of modified differential evolution algorithm has been proposed by Kaelo and Ali [13]. In the original DE three vectors are chosen at random for mutation and the base vector is then chosen at random within the three. This has an exploratory effect but it slows down the convergence of DE. Also the original DE uses a fixed positive value for the scaling factor F in mutation. This has an effect of restricting the exploration. The first modification to DE is to replace the random base vector $X_a^{(G)}$ in the mutation rule (14) with the tournament best $X_{\text{tb}}^{(G)}$. From the three random vectors the best is used as the base vector and the remaining two are used to find the differential vector in (14). This process explores the region around each $X_{\text{tb}}^{(G)}$ for each mutated point. This maintains the exploratory feature and at the same time expedites the convergence [13]. Also, instead of using a fixed F throughout a run of DE, we use a random F in $[-1,-0.4] \cup [0.4,1]$ for each mutated point [13]. This random localization feature gradually transforms itself into the search intensification feature for rapid convergence when the points in the solution space form a cluster around the global minimizer. This version of DE is referred to as the differential evolution algorithm with random localization (DERL) [13].

5. MDE implementation for OPF

The chromosome structure of MDE used for solving OPF is shown in Fig. 1. It is worth mentioning that the control variables are self-constrained. In order to keep the trial vectors within their bounds, the control parameter that exceeds a feasible bound is adjusted to the corresponding violated bound.

Generator MW output			Generator voltages			Transformer settings		
P_{G2}	P_{G3}	….	V_{G1}	V_{G2}	….	T_1	T_2	….

Fig. 1. Chromosome structure of MDE.

To handle the inequality constraints of state variables, including slack bus real and reactive power, load bus voltage magnitudes and transmission line loading, the extended objective function (fitness function) can be defined as

$$F = \sum_{i=1}^{NG} f_i + K_p(P_{G1} - P_{G1}^{\text{lim}})^2 + K_Q(Q_{G1} - Q_{G1}^{\text{lim}})^2 + K_V \sum_{i=1}^{NL} (V_{Li} - V_{Li}^{\text{lim}})^2 + K_S \sum_{i=1}^{NB} (S_{li} - S_{li}^{\text{lim}})^2 \tag{18}$$

where K_p, K_Q, K_V and K_S are the penalty factors, and x^{lim} is the limit value of the dependent variable x given as:

$$x^{\text{lim}} = \begin{cases} x^{\max} & \text{if } x > x^{\max} \\ x^{\min} & \text{if } x < x^{\min} \end{cases} \tag{19}$$

It should be noted that the constraints on the reactive power at each generator excluding slack bus are not included in the fitness func-

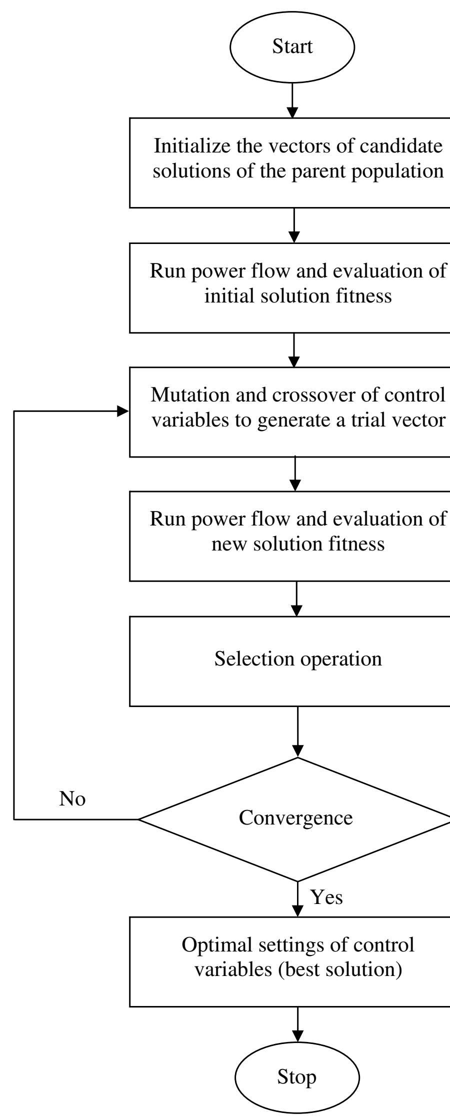

Fig. 2. General flowchart of the MDE-based OPF (MDE–OPF) solution algorithm.

tion (18). These constraints will be handled in the power flow algorithm.

The flowchart of OPF solution via MDE is shown in Fig. 2. The power flow algorithm is applied for each candidate solution to evaluate its fitness and determine the state variables. The optimization procedure stops whenever a predetermined maximum number of generations G^{max} is reached, or whenever the best candidate solution in the population does not improve over a predetermined number of generations.

6. Test results

In order to illustrate the efficiency and robustness of the proposed MDE-based OPF (MDE–OPF) algorithm, two case studies were performed. In the first case study, we consider the six-bus test system described in [14], with a quadratic model of generator cost curves. In the second case study, we consider the IEEE 30-bus system given in [15], with three different types of generator cost curves which are: a quadratic model in Case 2.1, a piecewise quadratic model in Case 2.2 and a quadratic model with sine component in Case 2.3. In each case study, two sets of 20 test runs for solving the OPF problem, were performed; the first set (DE–OPF) is based on the classical differential evolution algorithm and the second one (MDE–OPF) is based on the modified differential evolution algorithm. For the power flow convergence, the tolerance was set to 10^{-4} p.u.

Each optimization approach (DE–OPF and MDE–OPF) was implemented under the MATLAB computational environment, on a personal computer with Intel Pentium IV 3.0 GHz processor and 512 MB total memory.

6.1. Case study 1: six-bus test system

The six-bus system consists of 7 transmission lines and 4 generating units [3,14]. The generator cost curves are modeled by quadratic functions as:

$$f_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{20}$$

where a_i, b_i and c_i are the cost coefficients of the ith generating unit.

Bus 1 is considered as the swing bus. The limits assigned to each variable are [3,9]:

- voltage magnitudes: 0.95–1.05 p.u. for generation buses and 0.9–1.1 p.u. for load buses;
- unit active powers: 50–250 MW;
- unit reactive powers: ±75% of the unit maximum active power (equal to a 0.8 unit power factor);
- line loadings: 100 MVA for all lines, except line (4–5) whose limit is 50 MVA.

The control parameter settings of MDE–OPF algorithm are given in Table 1. The average cost obtained using MDE–OPF was 7872.323$/h with the minimum being 7872.256$/h and a maximum of 7872.620$/h (0.005% difference). The average execution time was 3.83 s. Also, it is important to point out that for all the trial runs, the convergence was reached without any violation of the constraints. Tables 2 and 3 present the power flow solution for the trial run that generated the minimum cost solution, which converged after 140 generations and 4.00 s. The convergence of MDE–OPF for the trial run with minimum cost solution is shown in Fig. 3.

The best solution obtained using DE–OPF are tabulated in Table 4, with the following parameter settings: $F = 0.70$, $C_R = 0.75$, $N_P = 16$ and $G^{max} = 160$. The comparison of the convergence characteristics is depicted in Fig. 3. It is clear that MDE–OPF and DE–OPF converge practically to the same solution, but MDE–OPF has considerably a faster convergence rate compared to DE–OPF.

The best solution obtained with MDE–OPF has been also compared with the results of classical economic dispatch and standard load flow (ED + LF), those published by Weber [14] and those reported using genetic algorithm (OPFGA) [3] and simulated annealing (OPFSA) [9]. The comparison of results is summarized in Table 4. It is clear that MDE approach gives a better global optimum solution with less computation time than the other techniques. These results clearly demonstrate the ability of the proposed approach to find the least expensive OPF solution, with minimum computation time than the classical and non-classical optimization approaches.

Table 1
Control parameter settings of MDE–OPF algorithm for case study 1

Parameter	Setting
Population size (N_P)	16
Crossover constant (C_R)	0.75
Maximum number of generations (G^{max})	160
Penalty factor of slack bus active power (K_P)	100
Penalty factor of slack bus reactive power (K_Q)	100
Penalty factor of voltage magnitudes (K_V)	50
Penalty factor of transmission line loadings (K_S)	8000000

Table 2
Power flow solution of the six-bus test system (case study 1)

Bus	Voltage (p.u.)	Angle (degree)	Generation		Load	
			MW	MVAr	MW	MVAr
1	1.0483	0.0	123.971	62.438	100	20
2	1.0500	2.079	195.269	3.587	100	20
3	1.0174	−3.240	109.187	53.964	100	20
4	1.0277	−0.150	179.106	20.414	100	20
5	1.0049	−2.244	0.0	0.0	100	50
6	0.9982	−3.844	0.0	0.0	100	10

Table 3
Line power flows of the six-bus test system (case study 1)

Line	From	To	P (MW)	Q (MVAr)	Transmission losses	
					MW	MVAr
1	1	2	−40.456	17.807	0.726	−0.750
2	1	5	64.427	24.631	1.752	1.395
3	2	4	54.087	2.143	1.065	−0.028
4	3	5	−11.309	20.771	0.233	−1.578
5	3	6	20.496	13.193	0.241	−1.550
6	4	5	49.817	4.249	0.951	−0.165
7	4	6	82.311	−1.664	2.566	3.079
Total losses					7.533	0.403

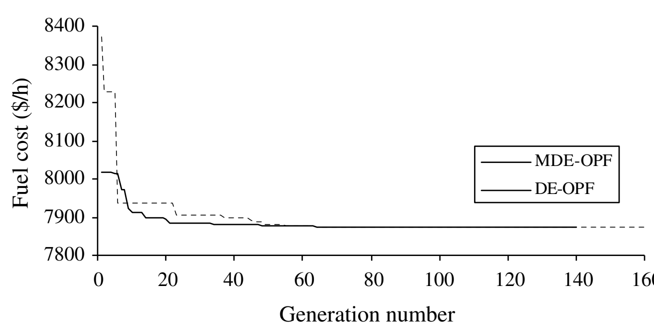

Fig. 3. Convergence of MDE–OPF and DE–OPF for case study 1.

Table 4
Comparison of different OPF methods for the six-bus test system (case study 1)

	ED + LF	Weber [14]	OPFGA [3]	OPFSA [9]	DE–OPF	MDE–OPF
P_{G1} (MW)	99.74	160.39	152.3252	131.80	123.366	123.971
P_{G2} (MW)	216.17	133.00	151.6563	190.98	195.074	195.269
P_{G3} (MW)	50.00	143.00	118.0913	109.15	109.818	109.187
P_{G4} (MW)	250.00	169.00	187.0893	178.24	179.240	179.106
Total generation (MW)	615.91	605.78	609.1621	610.17	607.498	607.533
Fuel cost ($/h)	7860.00	8062.00	7987.1764	7938.00	7872.300	7872.256
Transmission losses (MW)	15.91	5.38	9.2088	8.83	7.498	7.533
CPU time (s)	–	–	31	26	4.37	4.00
Violating quantities	2	0	0	0	0	0

6.2. Case study 2: IEEE 30-bus test system

The IEEE 30-bus system consists of 41 transmission lines, 6 generating units and 4 tap-changing transformers [15]. The minimum and maximum limits on control variables are shown in Table 7. In all cases, bus 1 is considered as the swing bus. Three different types of generator cost curves which are: a quadratic model, a piecewise quadratic model and a quadratic model with sine component have been considered as follows:

6.2.1. Case 2.1: Quadratic cost curve model

In this case the fuel cost characteristics for all generating units are modeled by quadratic functions given by (20). The generator cost coefficients are given in Table 5. The control parameter settings of MDE–OPF algorithm are shown in Table 6. The average cost found by MDE–OPF was 802.382$/h with a minimum cost of 802.376$/h, and a maximum cost of 802.404$/h (0.004% difference). Table 7 presents a summary of the optimization results for the solution with minimum cost, which converged after 120 generations and 23.25 s. Fig. 4 shows the convergence of MDE–OPF for the trial run that produced the minimum cost solution. It is important to note that all control and state variables remained within their permissible limits.

The best solution obtained from DE–OPF is shown in Table 8, with the following parameter settings: $F = 0.70$, $C_R = 0.70$, $N_P = 18$ and $G^{max} = 160$. The convergence characteristics are depicted in Fig. 4. Once again MDE–OPF appears to be superior to DE–OPF with regard to the solution quality and convergence rapidity.

The results of the proposed approach were compared in Table 8 to those reported using non-linear programming (NLP) [15], evolutionary programming (EP) [5], simulated annealing (SA) [9], tabu search (TS) [7] and improved evolutionary programming (IEP) [16]. It can be seen that the results given by MDE–OPF are better than those reported in the literature, except for the solutions of TS and SA reported in Abido [7] and Roa-Sepulveda [9], respectively, which are less expensive.

However, it is important to note that the best solution given in Abido [7] violates the slack bus lower Q-limit by 1.66 MVAr. It should be noted also that the optimal solution given in Roa-Sepulveda [9] was subject to a power mismatch tolerance of 0.01 p.u. that we judge insufficient.

Table 6
Control parameter settings of MDE–OPF algorithm for case study 2

Parameter	Setting value		
	Case 2.1	Case 2.2	Case 2.3
Population size (N_p)	18	20	20
Crossover constant (C_R)	0.50	0.85	0.75
Maximum number of generations (G^{max})	160	160	150
Penalty factor of slack bus active power (K_P)	100	100	500
Penalty factor of slack bus reactive power (K_Q)	100	100	500
Penalty factor of voltage magnitudes (K_V)	100,000	10,000	10,000
Penalty factor of transmission line loadings (K_S)	50	50	200,000

Table 7
Optimization results of MDE–OPF algorithm for case study 2

Variable	Limits		Case 2.1	Case 2.2	Case 2.3
	Lower	Upper			
P_{G1} (MW)	50	200	175.974	140.000	197.426
P_{G2} (MW)	20	80	48.884	55.000	52.037
P_{G5} (MW)	15	50	21.510	24.000	15.000
P_{G8} (MW)	10	35	22.240	34.989	10.000
P_{G11} (MW)	10	30	12.251	18.044	10.001
P_{G13} (MW)	12	40	12.000	18.462	12.000
V_{G1} (MW)	0.95	1.05	1.0500	1.0500	1.0371
V_{G2} (p.u.)	0.95	1.10	1.0382	1.0400	1.0130
V_{G5} (p.u.)	0.95	1.10	1.0113	1.0139	0.9648
V_{G8} (p.u.)	0.95	1.10	1.0191	1.0259	1.0320
V_{G11} (p.u.)	0.95	1.10	1.0951	1.0940	1.0982
V_{G13} (p.u.)	0.95	1.10	1.0837	1.0773	1.0890
T_{11}	0.90	1.10	0.9866	0.9714	1.0969
T_{12}	0.90	1.10	0.9714	1.0046	1.0909
T_{15}	0.90	1.10	0.9972	0.9902	1.0991
T_{36}	0.90	1.10	0.9413	0.9494	1.0021
Fuel cost ($/h)			802.376	647.846	930.793
Transmission losses (MW)			9.459	7.095	13.064
CPU time (s)			23.25	36.48	43.01

In addition, it can be seen that the computing time of the proposed MDE–OPF algorithm is better than the other heuristic techniques. As a conclusion, we can say that MDE–OPF has the ability

Table 5
Generator cost coefficients in case 2.1

Bus	P_G^{min} (MW)	P_G^{max} (MW)	Q_G^{min} (MVAr)	Q_G^{max} (MVAr)	Cost coefficients		
					a	*b*	*c*
1	50	200	−20	200	0	2.00	0.00375
2	20	80	−20	100	0	1.75	0.01750
5	15	50	−15	80	0	1.00	0.06250
8	10	35	−15	60	0	3.25	0.00834
11	10	30	−10	50	0	3.00	0.02500
13	12	40	−15	60	0	3.00	0.02500

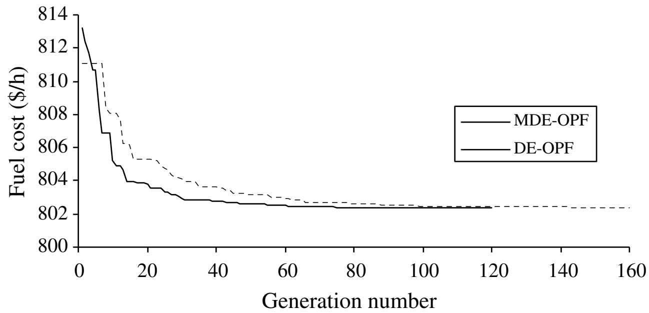

Fig. 4. Convergence of MDE–OPF and DE–OPF for case 2.1.

to find comparable or better solutions compared to those obtained with other heuristic approaches.

6.2.2. Case 2.2: piecewise quadratic cost curve model

The fuel cost characteristics for the generating units connected at bus 1 and 2 are now represented by a piecewise quadratic function to model different fuels [5]. The cost coefficients for these units are given in Table 9. The control parameter settings of MDE–OPF algorithm are shown in Table 6. The average cost was 648.356$/h with a minimum cost of 647.846$/h, and a maximum cost of 650.664$/h (0.44% difference). Table 7 presents a summary of the optimization results for the best solution, which converged after 160 generations and 36.48 s. The variation of the total fuel cost of the best solution is shown in Fig. 5.

Note also that all control and state variables remained within their permissible limits.

The best solution found by DE–OPF is shown in Table 10, with the following setup: $F = 0.40$, $C_R = 0.70$, $N_P = 20$ and $G^{max} = 160$. The comparison of the convergence characteristics is depicted in Fig. 5. It is quite clear that MDE–OPF give better results compared to DE–OPF.

For comparison purposes, the results reported using EP [5], PSO [8] and IEP [16] are shown in Table 10. It can be observed that the minimum cost solution obtained via MDE–OPF is better than the one given by IEP and close to those found by EP and PSO, with a reduced computation time. These results confirm the ability of the proposed method to find accurate OPF solutions in the presence of generating units with piecewise quadratic cost curve models.

6.2.3. Case 2.3: quadratic cost curve model with sine component

In this case, a sine component is added to the cost curves of the generating units at bus 1 and 2 to reflect the valve-point loading effects [17] as:

$$f_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 + |d_i \sin(e_i(P_{Gi}^{\min} - P_{Gi}))| \quad (21)$$

where a_i, b_i, c_i, d_i and e_i are the cost coefficients of the ith generating unit. The cost coefficients of the two units are given in Table 11.

Table 9
Generator cost coefficients for units 1 and 2 of case 2.2

Bus	Operating region		Cost coefficients		
	From (MW)	To (MW)	*a*	*b*	*c*
1	50	140	55.0	0.70	0.0050
	140	200	82.5	1.05	0.0075
2	20	55	40.0	0.30	0.0100
	55	80	80.0	0.60	0.0200

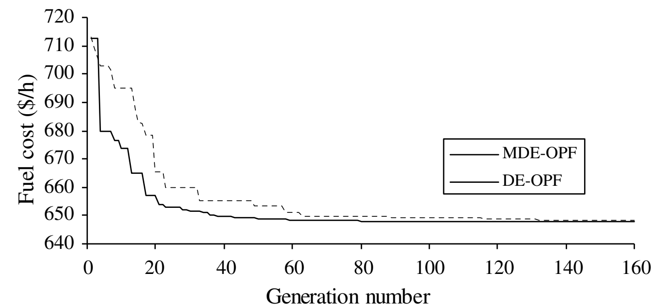

Fig. 5. Convergence of MDE–OPF and DE–OPF for case 2.2.

Table 10
Comparison of different OPF methods for IEEE 30 bus system (case 2.2)

	EP[5]	PSO[8]	IEP[16]	DE–OPF	MDE–OPF
P_{G1} (MW)	140.000	140.00	139.9962	139.961	140.000
P_{G2} (MW)	55.000	55.000	54.9849	54.984	55.000
P_{G5} (MW)	24.165	24.15	23.2558	23.910	24.000
P_{G8} (MW)	35.000	35.00	34.2794	34.291	34.989
P_{G11} (MW)	18.773	18.51	17.5906	21.161	18.044
P_{G13} (MW)	17.531	17.79	20.7012	16.202	18.462
Total generation (MW)	290.469	290.45	290.8081	290.509	290.495
Cost ($/h)	647.79	647.69	649.312	648.384	647.846
Losses (MW)	–	–	–	7.109	7.095
Average CPU time (s)	51.6	–	602.56	37.14	37.05

The control parameter settings of MDE–OPF algorithm are also shown in Table 6. The average cost was 942.501 $/h. The minimum and maximum costs found were 930.793 $/h and 954.073 $/h, respectively (2.5% difference). Table 7 shows the solution details for a minimum cost, which converged after 150 generations and 43.01 s. It is important to point out that all control and state variables were able to stay within their permissible limits. The convergence of MDE–OPF for the trial run with the minimum fuel cost is shown in Fig. 6.

The best solution found by DE–OPF is shown in Table 12, with the following parameter settings: $F = 0.70$, $C_R = 0.70$, $N_P = 20$ and $G^{max} = 150$. The convergence characteristic of DE–OPF is depicted

Table 8
Comparison of different OPF methods for IEEE 30 bus system (case 2.1)

	NLP [15]	EP [5]	SA [9]	TS [7]	IEP [16]	DE–OPF	MDE–OPF
P_{G1} (MW)	176.26	173.848	173.15	176.04	176.2358	176.009	175.974
P_{G2} (MW)	48.84	49.998	48.54	48.76	49.0093	48.801	48.884
P_{G5} (MW)	21.51	21.386	19.23	21.56	21.5023	21.334	21.510
P_{G8} (MW)	22.15	22.630	12.81	22.05	21.8115	22.262	22.240
P_{G11} (MW)	12.14	12.928	11.64	12.44	12.3387	12.460	12.251
P_{G13} (MW)	12.00	12.000	13.21	12.00	12.0129	12.000	12.000
Total generation (MW)	292.90	292.79	278.58	292.85	292.9105	292.866	292.859
Cost ($/h)	802.40	802.62	799.45	802.29	802.465	802.394	802.376
Losses (MW)	9.48	–	9.20	–	–	9.466	9.459
Average CPU time (s)	–	51.4	760	–	594.08	36.61	23.07

Table 11
Generator cost coefficients for units 1 and 2 of case 2.3

Bus	P_G^{min} (MW)	P_G^{max} (MW)	Q_G^{min} (MVAr)	Q_G^{max} (MVAr)	Cost coefficients				
					a	*b*	*c*	*d*	*e*
1	50	200	−20	200	150.0	2.00	0.0016	50.00	0.0630
2	20	80	−20	100	25.0	2.50	0.0100	40.00	0.0980

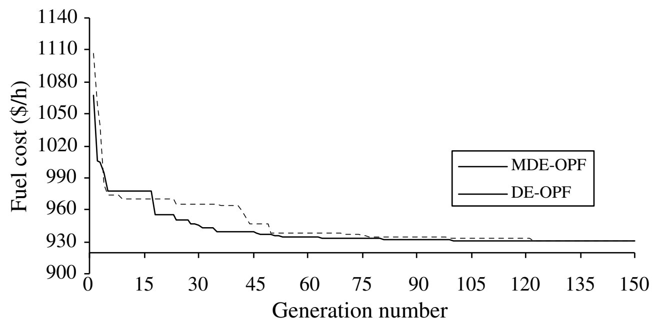

Fig. 6. Convergence of MDE–OPF and DE–OPF for case 2.3.

Table 12
Comparison of different OPF methods for IEEE 30 bus system (case 2.3)

	EP [5]	TS [7]	IEP [16]	DE–OPF	MDE–OPF
P_{G1} (MW)	199.600	200.00	149.7331	196.989	197.426
P_{G2} (MW)	20.000	39.65	52.0571	51.995	52.037
P_{G5} (MW)	22.204	20.42	23.2008	15.000	15.000
P_{G8} (MW)	24.122	12.47	33.4150	10.006	10.000
P_{G11} (MW)	14.420	10.00	16.5523	10.015	10.001
P_{G13} (MW)	13.001	12.00	16.0875	12.000	12.000
Total generation (MW)	297.877	294.54	291.0458	296.005	296.464
Cost ($/h)	919.89	919.72	953.573	931.085	930.793
Losses (MW)	–	–	–	12.605	13.064
Average CPU time (s)	–	–	–	44.96	41.85

in Fig. 6. It is evident that MDE–OPF outperforms DE–OPF in terms of solution quality and convergence rapidity.

The best solution found by MDE–OPF were compared in Table 12 with the results reported using EP [5], TS [7] and IEP [16]. It is clear that MDE–OPF outperforms IEP technique. On the other hand, the best fuel costs of EP and TS reported in Yuryevich and Wong [5] and Abido [7] are less expensive than that found by MDE–OPF. However, it should be noted that the best solution given in Yuryevich and Wong [5] violates the limits of Q_{G1} and line (1–2) loading by −252.04% and +17.0%, respectively [16]. The best solution given in Abido [7] also violates the limit of the line (1–2) loading by +4.1% [16]. These results confirm the ability of the proposed approach to find accurate and feasible OPF solutions in the presence of generating units whose fuel cost curves consider the effects of multiple valve-points.

7. Conclusion

In this paper, a modified differential evolution (MDE) algorithm has been introduced and applied to solve the OPF problem in the presence of generators with non-smooth and non-convex fuel cost curves. An improvement of the original DE algorithm was accomplished with a modification in mutation rule that enhances its rate of convergence without compromising solution quality. Simulation results show that MDE is superior to the original DE algorithm with regard to the rapid convergence to the exact global optimum.

The proposed approach was successfully and effectively implemented to find the optimal settings of the control variables of a 6-bus and the IEEE 30-bus test systems. The comparison of the results using the proposed approach to those reported in the literature; confirms its effectiveness and superiority to find accurate and feasible OPF solutions without any restrictions on the shape of the fuel cost curves.

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