Permanent magnet synchronous motor fault detection and isolation using second order sliding mode observer

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***Abstract*—This paper deals with the design of fault detection and isolation (FDI) scheme on the basis of high order sliding mode observer for permanent magnet synchronous motors(PMSM). More precisely, the main advantage of the use of high order sliding mode techniques is that it allows to avoid the chattering phenomenon which is inherent to the classical first order sliding mode observers and controllers. The efficiency of the proposed observers is illustrated on a Matlab/Simulink simulator.**

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are very suitable for modern industrial applications. As a matter of fact, their high efficiency, high torque, compactness and low weight place, it is in the lead of rotating electrical machines. Nonetheless PMSM are very sensitive to the strict constraints due to the environment of embarked systems. To improve the performance of PM synchronous motor, it is very important to design a controller with the fault tolerant and detected. In recent years, faut detection and isolation (FDI) has been studied extensively[1], [2], [3]. Advances in control theory have greatly sped up the development of FDI of dynamical systems and various approaches have been proposed. Among these approaches, observer based FDI is an effective one and has been widely studied especially in last decades. The fundamental purpose of a FDI scheme is to generate an alarm when a fault occurs and also, if it is possible, to locate the fault or to estimate its magnitude[4].

The concept of sliding modes was emerged from the Soviet Union in the sixties, where the effects of introducing discontinuous control action into dynamical systems were explored[14]. Beside the control context, sliding mode techniques are also used for observation and fault detection aims[4]. With the help of sliding mode techniques, it is possible to estimate the magnitude of a fault. This may very helpful for the design of fault tolerant control strategies. However, first order sliding mode exist the well-known chattering phenomenon which can have a big negative influence in fault reconstruction. To avoid this phenomenon high-order sliding modes can be used. As a consequence, observer for fault detected using high order sliding mode will offer good potential in the field of FDI. The application of such techniques to the FDI problem constitutes the main contribution of this paper.

The paper is organized as follows. Section II describes the model of synchronous motor with the fault. Section III mainly recall the principle of high order sliding mode and control algorithm. Section IV designs observer controller for detecting fault. In last section V, The simulation results are carried out in order to show the effectiveness of fault tolerant control and estimation of its magnitude.

II. SYSTEM MODEL

The electrical and mechanical equation of a 3-phase permanent magnet synchronous motor can be

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depicted in the so-called (d,q)-frame by application of the Park transformation and described by [5], [7].

$$\begin{aligned}
\frac{d\theta}{dt} &= \omega \\
\frac{d\omega}{dt} &= -\frac{P}{J}[(L_d - L_q)i_d + \phi_f]i_q - \frac{f_v}{J}\omega - \frac{C_l}{J} \\
\frac{di_d}{dt} &= -\frac{R_s}{L_d}i_d + P\frac{L_q}{L_d}\omega i_q + \frac{1}{L_d}u_d \\
\frac{di_q}{dt} &= -P\frac{\phi_f}{L_q}\omega - P\frac{L_d}{L_q}\omega i_d - \frac{R_s}{L_q}i_q + \frac{1}{L_q}u_q
\end{aligned} \tag{1}$$

where θ is the angular position of the motor shaft, ω is the angular velocity of the motor shaft, i_d is the direct current and i_q is the quadrature current. ϕ_f is the flux of the permanent magnet, P is the number of pole pairs, R_s is the stator windings resistance, L_d and L_q are the direct and quadrature stator inductances respectively. J is the rotor moment of inertia, f_v is the viscous damping coefficient and C_l is the load torque. u_d is the direct voltage and u_q is the quadrature voltage.

Let x denote the state $x = [\theta, \omega, i_d, i_q]^T$ and u the input $u = [u_d, u_q]^T$. Then, a state space model of the synchronous motor can be written as the following nonlinear system:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + G(x,t) + Bu(t) \\
y(t) &= Cx(t)
\end{aligned} \tag{2}$$

when the motor exist the internal fault, the fault model[16] of synchronous motor can be rewritten as:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + G(x,t) + Bu(t) + Df(u,t) \\
y(t) &= Cx(t)
\end{aligned} \tag{3}$$

Assume that the matrices D is full column rank and D represents a distribution matrix. The function $f(t)$ is unknown but bounded so that

$$\|f(u,t)\| \le \sigma(u,t) \tag{4}$$

where $\sigma(u,t)$ is a known function. The signal $f(t)$ represents internal fault. where, matrix A,B and C are constant coefficient matrices,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{f_v}{J} & 0 & \frac{P\phi_f}{J} \\ 0 & 0 & -\frac{R_s}{L_d} & 0 \\ 0 & -\frac{P\phi_f}{L_q} & -\frac{R_s}{L_q} & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$G(x,t)$ is assumed as the nonlinear term.

$$G = \begin{bmatrix} 0 \\ P(L_d - L_q)i_d i_q/J \\ PL_q\omega i_q/L_d \\ PL_d\omega i_d/L_q \end{bmatrix}$$

All the functions are assumed to be continuous in their arguments.

III. HIGH ORDER SLIDING MODE

To reducer or to avoid the chattering problem, high order sliding mode can be used. Emel'yanov et al. initially presented the idea of acting on the higher derivatives of the sliding variable and provided second order sliding algorithms such as the Twisting Algorithm, and algorithm with a prescribed law of convergence[8]. The so-called Super-Twisting Algorithm, which is applicable to systems with the relative degree one with respect to the sliding variable[9], completely removes chattering. Levant reported that in second order sliding, the sliding accuracy is proportional to the square of the switching time delay which turns out to be another advantage of higher order sliding modes[10].

Without loss of generality, consider a nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{5}$$

where, the state vector $x \in \mathbb{R}^n$, the control $u \in \mathbb{R}$ and $f(x)$,$g(x)$ are smooth vector functions of proper dimension. The control u is determined by feedback control $u = U(t,x)$, where U is possibly a discontinuous function.

Define a candidate sliding surface $s \in \mathbb{R}$ as

$$s = S(t,x) \tag{6}$$

such that by making it zero, the control objective is fulfilled. $S(\cdot)$ is a sufficiently smooth constraint function.

If the system in equation (5) has a globally defined relative degree r[11] with respect to the sliding variable s defined in equation (6), then the sliding surface dynamics can be expressed in $I-O$ form as

$$\begin{aligned} s^{(r)} &= L_f^r S(x) + L_g L_f^{r-1} S(x) u \\ &= \phi(t,x) + \gamma(t,x) \end{aligned} \tag{7}$$

where L_g, L_f are Lie derivatives, $L_g L_f^{r-1} S(x) \neq 0$ holds globally by assumption and the discontinuity does not appear in the first $(r-1)$ total time derivatives of the constraint variable s along the trajectories, i.e., $s, \dot{s}, \ddot{s}, \cdots, s^{(r-1)}$ exist and are single valued.

The resulting zero dynamics of order $n-r$ can be written as

$$\dot{\eta} = \mathcal{Z}(\hat{s}, \eta) \tag{8}$$

where, η satisfy the condition

$$L_g \eta_i(x) = 0; \quad i = 1, 2, \cdots, (n-r) \tag{9}$$

and $\hat{s} = [s, \dot{s}, \ddot{s}, \ldots, s^{(r-1)}] = [s, L_f s, L_f^2 s, \ldots, L_f^{r-1} s]$.

Assumption 1: The zero dynamics (8) is input-to-state stable i.e., for any bounded $\hat{s}(t)$, the internal states $\eta(t)$ remain bounded. For $r = n$, there are no zero dynamics and the system is said to be fully linearizable.

Now the task is to find a feedback control u such that the nonlinear uncertain dynamics (7) can be stabilized on the basis of suitable known upper bounds to the uncertainties appearing in (7). In general, any sliding mode controller that keeps $s = 0$ needs $s, \dot{s}, \ddot{s}, \cdots, s^{(r-1)}$ to be made available. It must be stressed that while the simple discontinuous relay control on $s = 0$ is effective when $r = 1$, it is possibly unstable if $r = 2$ and always unstable for $r > 2$ or at most produces a stable periodic motion[12].

Definition 1: Given the constraint (6), its r^{th} order sliding set s^r is defined by r equalities

$$s = \dot{s} = \ddot{s} = \cdots = s^{(r-1)} = 0 \tag{10}$$

which constitute an $r-$dimensional condition on the system states. According to this definition, if the only imposed condition on the sliding motion is that of $s = 0$, then the sliding set S is said to be of first order which corresponds to the case of standard sliding mode control.

Definition 2: Assume the $r^{th}-$order sliding set (10) is nonempty and assume that it consists of Filippovs trajectories of the discontinuous dynamical system (5). Then the corresponding motion satisfying (10) is termed a $r^{th}-$order sliding motion with respect to the constraint function s.

To achieve (10), there must exist constant s_0,Γ_m,Γ_M and Φ such that system (6) satisfy

$$\begin{aligned} &|\varphi(x,t)| \leq \Phi \\ &|s| \leq s_0 \\ &0 < \Gamma_m \leq |\gamma(x,t)| \leq \Gamma_M \end{aligned} \tag{11}$$

A. Twisting Algorithm

This algorithm[14] is characterized by a twisting of the phase portrait around the origin. The finite time convergence to the origin is due to switching between two different control amplitudes as the trajectory comes nearer to the origin. The sign of the derivative of the sliding variable is required for decision making.Let the relative degree of the system to be 1, the control algorithm is defined by the following control law[15]:

$$\dot{u}(t) = \begin{cases} -u & |u| > u_{max} \\ -V_m sign(s) & s\dot{s} \leq 0; |u| \leq u_{max} \\ -V_M sign(s) & s\dot{s} > 0; |u| \leq u_{max} \end{cases} \tag{12}$$

where, V_m,V_M,u_{max} are constant, and the corresponding sufficient conditions for the finite time convergence to the sliding manifold are[8]:

$$\begin{cases} 0 < V_m < V_M \\ \frac{\Phi}{\Gamma_m} < V_m \\ \Gamma_m V_M - \Phi > \Gamma_M V_m + \Phi \end{cases} \tag{13}$$

B. Super Twisting Algorithm

In this algorithm the trajectories on the second order sliding plane are also characterized by twisting around the origin. But this algorithm is just developed to control systems with relative degree 1 in order to avoid chattering. The continuous control law u consists of two terms. The first is defined by means of its discontinuous time derivative, while the other is a continuous function of the available sliding variable. The super twisting algorithm has the advantage that it needs not the time derivative of the sliding manifold.

The algorithm is defined by the following control law[15]:

$$\begin{aligned} u(t) &= u_1(t) + u_2(t) \\ \dot{u}_1 &= \begin{cases} -u & |u| > u_{max} \\ -W sign(s) & |u| \leq u_{max} \end{cases} \\ u_2 &= \begin{cases} -\lambda |s_0|^{\rho} sign(s) & |s| > s_0 \\ -\lambda |s|^{\rho} |sign(s) & |s| \leq s_0 \end{cases} \end{aligned} \quad (14)$$

Besides of the conditions (13), system also must satisfy:

$$\begin{cases} W > \frac{\Phi}{\Gamma_m} \\ \lambda^2 \geq \frac{4\Phi\Gamma_M(W+\Phi)}{\Gamma_m^3(W-\Phi)} \\ 0 < \rho \leq 0.5 \end{cases} \quad (15)$$

For $\rho = 1$, the algorithm converges to the origin exponentially. For systems where $s_0 = \infty$ and there is no bound on the control, the algorithm can be simplified as:

$$\begin{aligned} u(t) &= -\lambda |s|^{\rho} sign(s) + u_1(t) \\ \dot{u}_1 &= -W sign(s) \end{aligned} \quad (16)$$

IV. DESIGN OF THE OBSERVER

The sliding mode observer is designed for estimating the fault in synchronous motor. Dynamic nonlinear system (3) with fault is the following observer associated.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + G(\hat{x}, t) + Bu(t) + Lv \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \quad (17)$$

The variable v will be replaced with a sliding mode control law like (12) or (14), L is the feedback gain.

Let $e = \hat{x} - x$, observer function (17) subtracts system state function (3), it can obtain:

$$\dot{e} = Ae + G(\hat{x}, t) - G(x, t) + Lv - Df \quad (18)$$

There exists a change of coordinates T satisfies:

$$\begin{bmatrix} e_1 \\ e_y \end{bmatrix} = Te, \quad \begin{bmatrix} 0 \\ L_2 \end{bmatrix} = TL, \quad \begin{bmatrix} 0 \\ D_2 \end{bmatrix} = TD$$

So that (18) can be rewritten as:

$$\begin{aligned} \dot{e}_1 &= A_{11}e_1 + A_{12}e_y + G_1(T^{-1}\hat{x}, t) - G_1(T^{-1}x, t) \\ \dot{e}_y &= A_{21}e_1 + A_{22}e_y + G_2(T^{-1}\hat{x}, t) - G_2(T^{-1}x, t) \\ &+ L_2 v - D_2 f \end{aligned} \quad (19)$$

When the sliding occurs, $e_y = \dot{e}_y = 0$, so it can be obtain the form flowing:

$$\begin{aligned} \dot{e}_1 &= A_{11}e_1 + G_1(T^{-1}\hat{x}, t) - G_1(T^{-1}x, t) \\ 0 &= A_{21}e_1 + G_2(T^{-1}\hat{x}, t) - G_2(T^{-1}x, t) \\ &+ L_2 v - D_2 f \end{aligned} \quad (20)$$

According to the Lispschitz definition of function, it follows:

$$\begin{aligned} ||G_1(T^{-1}\hat{x}, t) - G_1(T^{-1}x, t)|| &\leq L_{G_1}||T^{-1}|| \cdot ||e_1|| \\ ||G_2(T^{-1}\hat{x}, t) - G_2(T^{-1}x, t)|| &\leq L_{G_2}||T^{-1}|| \cdot ||e_1|| \end{aligned} \quad (21)$$

Where L_{G_1} and L_{G_2} are the constant of Lispschitz, and because of $lim_{t \mapsto \infty} e_1 = 0$, therefore form (20) becomes finally:

$$f = D_2^{-1} L_2 v \quad (22)$$

For the PM synchronous motor, the control objective is double aspect. First, the rotor angular position $x_1 = \theta$ must track a reference trajectory angular position x_{1ref}. Second, the nonlinear electromagnetic torque must be linearized to avoid reluctance effects and torque ripple. This objective is equivalent to constrain $x_3 = i_d$ to track a constant direct current reference $x_{3ref} = 0$. Then, assuming all the state variables are available for measurement. A first sliding manifold for the tracking of the estimated value of direct current $\hat{i}_d$ towards the real value of direct current i_d, so the sliding mode variable s_1 is defined by

$$e = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \hat{i}_d - i_d \\ \ddot{\hat{\theta}} - \ddot{\theta} + k_1(\dot{\hat{\theta}} - \dot{\theta}) + k_2(\hat{\theta} - \theta) \end{bmatrix} \quad (23)$$

Note that the relative degree of s_1 equal 1. To track the angular position θ, another sliding mode variable can be defined as the following form for a desired second order dynamic. where k_1 and k_2 are positive parameters such that $P(z) = \ddot{z} + k_1\dot{z} + k_2 z$ is Hurwitz polynomial. Note also that the relative degree of s_2 equals 1.

At Fig. 1, sliding mode observer is redescribed. The control law in system adopts super twisting algorithm, and the sliding mode variable is consists of error between estimation and real value. Finally, observer generates the fault estimated via a filter, which can be designed as $1/(0.05s + 1)$.

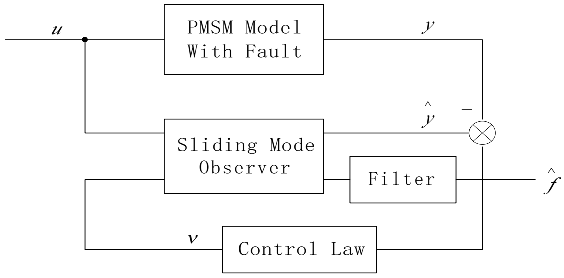

Fig. 1: Sliding Mode Observer

V. RESULT OF SIMULATION

This part displays the simulation results of high order sliding mode observer for fault detection and isolation. A PMSM is chosen to illustrate the performance which is a DutyMax 95DSC060300 (Leroy Somer Co.) drive. Two sensors give measurements of phase currents, a optical encoder is used to measure the position of the motor. The parameters of synchronous motor are shown in tab. I. A phase current of the maximum accepted value is $6.0A$, the load torque maximum value is $6N \cdot m$, and angular velocity is $3000rpm$.

From the knowledge of the nominal parameters and their variables, the controller parameter have been chosen to satisfy condition (13), (15), and to set the dynamic behavior of the system. It gives $k_1 = 200$, $k_2 = 100$, and in the super twisting algorithm, $\rho = 0.5$, $W = 0.8$, $\lambda = 0.5$.

TABLE I: The parameters of PMSM

R_s	3.3	Ω	J	0.037	$kg \cdot m^2$
L_d	0.027	H	f_v	0.0034	$N \cdot m \cdot s$
L_q	0.0339	H	P	3	−−
ϕ_f	0.341	Wb	Cl	2	$N \cdot m$

To achieve the efficiency of controller, at Fig. 2, it represents the detection signal $\hat{f}$ compared with the known fault signal $f = 0.1 + 0.1sin(2\pi t)$. In this figure, a good detection is obtained. Similarly,

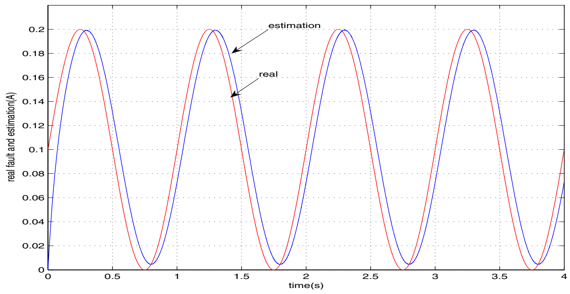

Fig. 2: Real Fault and Estimation

when the inject fault f like a square, Fig. 3 shows the nicer result of detecting the real fault.

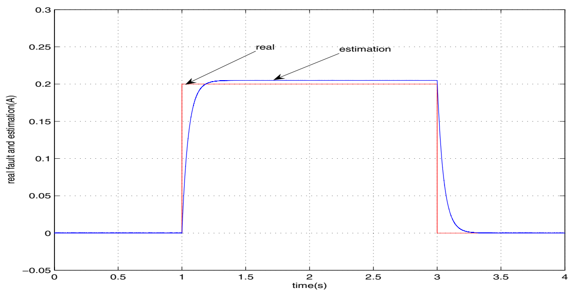

Fig. 3: Real Fault and Estimation

The state variables of PMSM are shown in Fig. 4. It can be seen that the trajectory of position is tracked and the current of direct axis is near to zero.

VI. CONCLUSION

The problem of detecting the internal fault in synchronous motor has been realized by second order sliding mode observer. Using this method, a good result of fault detection is obtained. More

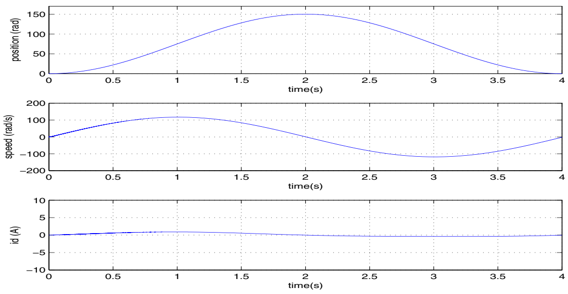

Fig. 4: State Variables of PMSM

precisely, second order sliding mode technique reduces or avoids the chattering phenomenon, despite it is quite more complex to design than first order sliding mode. This paper provides a straightforward method of the MIMO system for fault detection and isolation, the next step of this work will be the implementation of the designed observers on the PMSM.

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